A Modified Newton Method Formulation for Microwave Imaging

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Abstract—A new variant of Newton type methods has been developed for quantitative microwave imaging. To deal with the ill-posedness of the inverse problems, standard Newton type methods involve a linearization of the so called *data* equation using the Fréchet derivative with respect to the contrast function. Here, the formulation is expanded to include the *object* equation, therefore, the formulation seeks to reduce the errors in both the *data* and the *object* equations. While this modification does not remove the need to solve forward problem at each step, it nevertheless significantly improves convergence rate and the performance. To assess the efficiency of the proposed technique, numerical simulations with synthetic and experimental data have been carried out. The results demonstrate that the proposed variant outperforms the standard Newton method, and shows comparable performance to the contrast source inversion (CSI) algorithm with fewer iterations.

Index Terms—Newton methods, inverse scattering, microwave imaging, quantitative techniques

I. INTRODUCTION

Microwave imaging techniques aim to determine the material properties of an investigation domain via measurement of the scattered electromagnetic field. Therefore, they form a subgroup of electromagnetic inverse scattering problem, where the goal is to determine the shapes, locations and the electromagnetic parameters of unknown scatterers. Various solution techniques have been proposed in the literature to overcome the inherent ill-posedness of the problem, and to provide a solution.

These techniques can be grouped under two categories, namely qualitative and quantitative techniques. The qualitative techniques such as factorization method [1], or linear sampling [2], aim to reconstruct only the shapes and the locations of the scatterers. This limited aim enables one to form fast and easy to implement solution techniques, which are computationally cheaper and generally considered to be non-iterative. However, for a research field such as medical imaging, it is crucial to determine at least an approximation for the material parameters to differentiate biological tissues. Therefore, especially for medical imaging one needs a quantitative imaging technique which reconstructs not only the shape but also the electromagnetic parameters of the scatterers. That is, these algorithms aim to reconstruct the entire contrast profile of the investigation domain indicating material properties of the scatterers.

One of the most commonly used quantitative technique is the Newton method [3], [4]. In Newton method, the non-linear data equation, which is the integral equation for the scattered electromagnetic field, is linearized and then inverted via a regularization technique such as Tikhonov regularization. Like the Born iterative method [5] or distorted Born iterative method [6], this is an iterative technique that requires solving the forward scattering problem at each step to update the reconstructed scattered field. This is a computationally expensive operation, which is the main drawback of Newton type methods.

There might be two different approaches to overcome this limitation. The first one is to eliminate the need to solve forward problem by devising a new formulation. In the contrast source inversion (CSI) method, a cost function involving the error in data and object equations, object equation being the integral equation related to the total field inside the investigation domain, is formed and minimized via conjugate gradient technique [7]. The CSI method eliminates the need solve the forward problem, however, it has a relatively slower convergence rate.

The second approach to improve the standard Newton method might be to increase the convergence rate, and therefore to shorten the iterative process. To this end, in this work we propose a modified Newton type method which seeks to linearize not just the data equation, but a combination of the object and data equations. While this formulation still requires solving forward problem at each step, the additional information provided by the object equation significantly improves the efficiency of each iteration, and therefore, improves both the convergence rate and the performance of the Newton method. To demonstrate this, we analyzed an inverse scattering problem involving nested cylindrical scatterers, and reconstructed the contrast profile using three different methods, namely, CSI, standard Newton, and the modified version. The numerical simulations show that the variant developed here outperforms standard Newton method, and produces results similar to those of CSI, in fewer iterations. Finally, the method was tested with experimental Fresnel data presented in [8], to demonstrate that it can reconstruct unknown profiles via actual measurements of the scattered field.
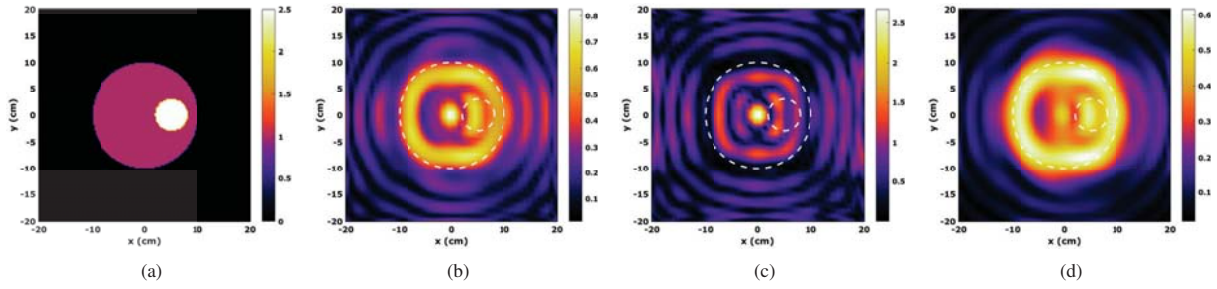
Fig. 1. (a) The exact contrast profile χ of the nested cylindrical scatterers. The contrast function reconstructed via three different algorithms, (b) Modified Newton in 13 iterations, (c) Standard Newton in 50 iterations, (d) Contrast source inversion in 50 iterations. The white dashed lines indicate the boundaries of the nested cylinders.

II. FORMULATION OF THE METHOD

Consider a domain D, called investigation domain, containing unknown scatterers. This region is illuminated by time-harmonic incident electromagnetic field E_s^{inc}, $s = 1, .., S$ denoting the source points. The scattered field E_s^{sct} is measured on a domain Γ outside D. Any forward or inverse scattering problem is based on the couple of integral equations known as the *object* and the *data* equations respectively:

$$E_s(r) = E_s^{inc}(r) + k_b^2 \int_D g(r,r')\chi(r')E_s(r')dr', \ r,r' \in D, \tag{1}$$

$$E_s^{sct}(r) = k_b^2 \int_D g(r,r')\chi(r')E_s(r')dr', \ r \in \Gamma, \ r' \in D. \tag{2}$$

Here, $E_s(r)$ denotes the total field inside D, and $g(r,r') = \frac{i}{4}H_0^{(1)}(k_b\|r-r'\|)$ is the free space Green's function. The term $\chi(r)$ is called as the contrast function, and it is defined as $\chi(r) = \frac{k^2(r)}{k_b^2(r)} - 1$, where $k(r) = \omega\sqrt{\epsilon_0\mu_0\epsilon_r(r)}$, and $\epsilon_r(r) = \frac{\epsilon(r)}{\epsilon_0} + i\frac{\sigma(r)}{\omega\epsilon_0}$ is the complex relative dielectric constant characterizing the scatterers. The goal of inversion algorithms is to reconstruct $\chi(r), r \in D$ from the measured $E_s^{sct}(r), r \in \Gamma$.

To demonstrate the modified Newton algorithm, let us first write (1) and (2) in compact form as operators operating on the contrast function:

$$L_1(\chi) = \chi E_s - \chi E_s^{inc} + \chi k_b^2 \int_D g(r,r')\chi(r')E_s(r')dr' = 0 \tag{3}$$

$$L_2(\chi) = k_b^2 \int_D g(r,r')\chi(r')E_s(r')dr' = E_s^{sct} \tag{4}$$

Note that (3) is obtained by multiplying both sides of (1) with $\chi(r)$. In standard Newton methods, only the non-linear operator $L_2(\chi)$ is linearized using the first two terms of Taylor series expansion around an initial guess for χ. Here, we use both (3) and (4) to formulate the linear equation that will be used to update χ iteratively:

$$L_1^F(\delta\chi) + L_2^F(\delta\chi) = E_s^{sct} - L_1(\chi) - L_2(\chi) \tag{5}$$

Here, $\delta\chi$ is the update amount for $\chi(r)$. The operators

$$L_1^F(\delta\chi) = \delta\chi E_s - \delta\chi E_s^{inc} + \delta\chi k_b^2 \int_D g(r,r')\chi(r')E_s(r')dr' \tag{6}$$

and

$$L_2^F(\delta\chi) = k_b^2 \int_D g(r,r')\delta\chi(r')E_s(r')dr' \tag{7}$$

are the Fréchet derivatives with respect to χ of $L_1(\chi)$ and $L_2(\chi)$ respectively. The algorithm starts with an initial guess for χ, and solves the forward problem to obtain $L_1(\chi)$ and $L_2(\chi)$. The update amount is then calculated by solving (5) via Tikhonov regularization

$$\delta\chi = \left(\alpha I + L^{F\star}L^F\right)^{-1} L^{F\star}\left(E_s^{sct} - L_1 - L_2\right) \tag{8}$$

where $L^F = L_1^F + L_2^F$ is the augmented matrix form of the Fréchet derivatives, and $\star$ denotes conjugate transpose. α is the regularization parameter. Finally, the function χ is updated via $\chi^{n+1} = \chi^n + \delta\chi^n$, n denoting number of iterations. The iterative process is continued until the ratio of ℓ^2 norms $\frac{\|\delta\chi^n\|}{\|\chi^n\|}$ becomes smaller than a predefined threshold ϵ_T.

III. NUMERICAL SIMULATIONS

To assess the practical applicability of the method, and to compare its performance with standard Newton and CSI methods, we first analyzed an inverse scattering problem involving nested cylindrical scatterers shown in Fig.1(a). The larger cylinder, which has a radius of 10 cm, is centered at the origin. The smaller cylinder is centered at the point $x = 5$ cm, $y = 0$ cm; and has a radius of 3 cm. The contrast values for the cylinders are chosen as $\chi = 1$, and $\chi = 2.5$ respectively. These contrast values are relatively high for a quantitative imaging application. However, we chose to analyze this configuration in order to demonstrate the capabilities and the limitations of the methods. Also, a small imaginary part is added to both contrast values in order to reflect conductivity losses. The scatterers are illuminated by 24 line sources located equiangularly on a circle with a radius of 40 cm. These points also act as receivers for the measurement of scattered field. The scattered field is produced via standard method of moments, and an SNR = 50 dB Gaussian noise is added to the result. The operating frequency is chosen as 2 GHz.
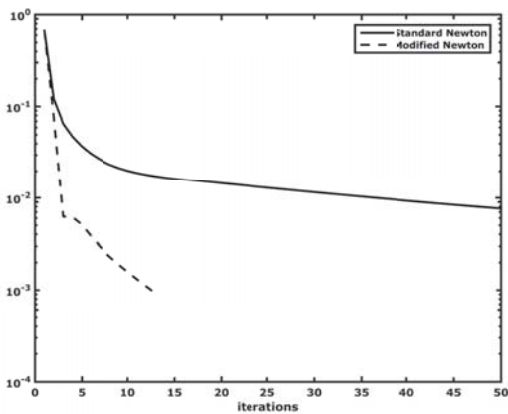
Fig. 2. The ratio of the norms $\frac{\|\delta\chi^n\|}{\|\chi^n\|}$ at each iteration in logarithmic scale for two version of Newton algorithms.

Fig. 3. Reconstructed relative dielectric constant ϵ_r for Fresnel data set FoamTwinDiel at 2 GHz.

For the inverse problem, the investigation domain of 40 cm × 40 cm is discretized into 42×42 points. The contrast function is reconstructed via three different algorithms; modified Newton, standard Newton, and CSI, with no a priori information. For all three methods, the initial guess for the contrast function is provided by the backpropagation algorithm [7]. The results are presented in Fig.1. Here, the stopping threshold for Newton methods is chosen as $\epsilon_T = 10^{-3}$. The convergence rates are given in Fig.2. The modified Newton algorithm presented in this paper converged in 13 iterations, whereas the standard Newton method did not reached the threshold within 50 iterations. CSI algorithm was also run for 50 iterations. As seen in Fig.1(b), the modified Newton algorithm accurately delineates the boundary of the larger cylinder, and approximates its contrast value. In this regard, it outperformed the results of the standard Newton method given in Fig.1(c). For the high contrast smaller cylinder, no method managed to reconstruct exact contrast value. However, the results of modified Newton and CSI at least indicate approximate position of this cylinder as seen in Fig.1(b) and in Fig.1(d). Overall, it can be concluded that the modified Newton algorithm produces outcomes comparable with that of CSI in fewer iterations, while significantly outperforming the standard version of Newton method.

Finally, to test the performance of the proposed method with experimental data, the Fresnel data set FoamTwinDiel presented in [8] has been used for reconstruction. As explained in [8], this data set provides the field scattered from two homogeneous dielectric cylinders of relative dielectric constant $\epsilon_r = 3$, for different operating frequencies. Here, the data for 2 GHz is used, and the relative dielectric constant ϵ_r is reconstructed via modified Newton algorithm. For this reconstruction, the investigation domain was discretized into 15×15 points. The result presented in Fig.3 shows that while the method failed to completely distinguish two cylinders for this disc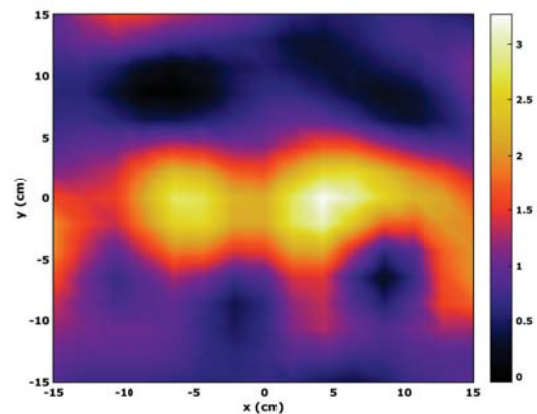retization, it nevertheless accurately reconstructed the value ϵ_r for the cylinders, which is the main strength of a quantitative imaging technique. This example suggests that the modified Newton method is suitable for working with experimental measurement data.

IV. Conclusion

In this study, a variant of the Newton type methods has been developed. The linearization process of the Newton method has been extended to include both object and the data equations, and the Fréchet derivatives are updated accordingly. This approach significantly improved the convergence rate, and produced more promising results for practical applications in microwave imaging. Although it is formulated for a 2-D problem in this paper, the formulation can be applied for 3-D problems in a straightforward manner.

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