

TURKISH PRIVATE PENSION FUND SIZE FORECASTING AS AN APPLICATION OF DATA ANALYTICS

Capstone Project

Serdar Ufuk Kara

İSTANBUL, 2020

MEF UNIVERSITY

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Advisor: Asst. Prof. Dr. Tuna Çakar

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MEF UNIVERSITY

Name of the project: Turkish Private Pension Fund Size Forecasting as an Application of Data Analytics

Name/Last Name of the Student: Serdar Ufuk Kara Date of Thesis Defense: 30/12/2020

I hereby state that the graduation project prepared by Serdar Ufuk Kara has been completed under my supervision. I accept this work as a "Graduation Project".

30/12/2020 Asst. Prof. Dr. Tuna Çakar

I hereby state that I have examined this graduation project by Serdar Ufuk Kara which is accepted by his supervisor. This work is acceptable as a graduation project and the student is eligible to take the graduation project examination.

30/12/2020

Director of the Graduate Program in Information Technologies

We hereby state that we have held the graduation examination of Serdar Ufuk Kara and agree that the student has satisfied all requirements.

THE EXAMINATION COMMITTEE

Committee Member

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1. Asst. Prof. Dr Tuna Çakar

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EXECUTIVE SUMMARY

TURKISH PRIVATE PENSION FUND SIZE FORECASTING AS AN APPLICATION OF DATA ANALYTICS

Serdar Ufuk Kara

Advisor: Asst. Prof. Dr. Tuna Çakar

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In this study univariate and multivariate models are used to forecast the net changes in total pension fund size of a private pension company in Turkey, using the daily data between November 2003 and November 2020. Univariate models include the naïve, autoregressive (AR), moving average (MA), and autoregressive moving average (ARMA) models. Multivariate models include vector autoregression (VAR) and multiple linear regression models. Our findings suggest that multivariate model predictions outperform univariate model predictions. Univariate model predictions can be improved with walk forward approach. Increased lag size can help improve AR, MA, ARMA and VAR model predictions. Naïve model produces the weakest predictions.

Key Words: pension fund size change, univariate model, multivariate model, autoregressive moving average model, vector autoregression model.

ÖZET

BİR VERİ ANALİTİĞİ UYGULAMASI OLARAK TÜRK BIREYSEL EMEKLİLİK FON BÜYÜKLÜKLERİ TAHMİNİ

Serdar Ufuk Kara

Proje Danışmanı: Dr. Öğr. Üyesi Tuna Çakar

ARALIK, 2020, 36 Sayfa

Bu çalışmada, Kasım 2003 ile Kasım 2020 arasındaki günlük veriler kullanılarak Türkiye'deki bir bireysel emeklilik şirketinin toplam emeklilik fon büyüklüğündeki net değişiklikleri tahmin etmek için tek değişkenli ve çok değişkenli modeller kullanılmıştır. Tek değişkenli modeller naif, özbağlanımlı (AR), kayan ortalamalı (MA) ve özbağlanımlı kayan ortalamalı (ARMA) modellerini içermektedir. Çok değişkenli modeller vektör özbağlanımlı (VAR) ve çoklu doğrusal regresyon modellerini içermektedir. Bulgularımız, çok değişkenli model tahminlerinin tek değişkenli model tahminlerinden daha iyi performans verdiğini göstermektedir. Tek değişkenli model tahminleri ileriye doğru yürüme yaklaşımı ile geliştirilebilmektedir. Arttırılmış gecikme boyutu, AR, MA, ARMA ve VAR modeli tahminlerini iyileştirmeye yardımcı olmaktadır. Naif model en zayıf tahminleri üretmektedir.

Anahtar Kelimeler: emeklilik fonu büyüklüğü değişimi, tek değişkenli model, çok değişkenli model, özbağlanımlı kayan ortalamalı model, vektör özbağlanımlı model.

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1. INTRODUCTION

Private Pension System (PPS) is a financial system that is built to support the future wealth of participants, in other words customers, in which they save in the earlier stages of active working years in order to be relatively better off in the years of retirement. Participation to the PPS can be voluntary, complementary, or compulsory. In PPS participants make periodic payments to their private pension accounts to benefit from some additional advantages compared to other instruments of savings and financial investments. Main advantages of savings through pension system are the government subsidies and tax benefits. In addition, participants have a chance to benefit from higher returns on investment through pension system where investments are managed by professional portfolio managers compared to most participants' individual efforts on financial investing, which requires some level of financial literacy. Pension accounts are likely to offer higher returns since the sources of the participants are gathered in funds, which offer the scale advantages in obtaining higher returns.

Participants mainly prefer to make payments in fixed amounts or fixed proportions of their salaries or define payment instructions for fixed amounts by their credit cards. Participant can also make additional irregular payments to their pension accounts through banking and other payment channels. These payments are directed to investment by private pension firms in various pension funds. Pension firms offer funds with different investment strategies to match the risk preference and return expectations of the participants. If the participant is highly risk averse, contribution payments are channeled to less risky funds (or a less risky mix of funds) mainly consists of debt instruments, money market instruments and time deposits, which offers lower variability and low to medium returns. Conversely, if the participant is a risk seeker, contribution payments are invested in more risks funds (or a more risky composition of funds) that invests in domestic common stocks, precious metals, foreign exchange denominated debt instruments, foreign common stocks, mutual funds, and more, which have higher volatility and offer higher returns. Depending on the risk profile of the participants.

Size of total retirement assets increase as more new savings are added to the system than the exits due to retirement, early withdrawal, or death of participants. Moreover, total assets under management increase due to investment returns of the pension funds. Funds make long term investments on companies that signals growth prospects in industries such as technology, transportations and telecommunications, along with traditional investments on banking, precious metals, and government and corporate financing.

According to OECD's Pensions Markets in Focus report, as of 2019 total retirement assets under management amounts more than 50 trillion USD globally. These savings are generally higher in developed countries as a percentage of GDP, such as in Denmark and Netherlands - about %200 of GDP, compared to developing countries, such as in Turkey and Greece – less than %10 of GDP [1]. In OECD's report, growth in asset size has been attributed to increasing share of working-age population and introduction of compulsory pension plans in some countries, whereas asset growth is discussed to be limited by the benefit payments of the system. In addition, remarkable growth of pension assets in 2019 is attributed to the "strong investment performance", mainly due to the good performance of stock exchange markets in 2019. Pension funds are reported to perform very well in real investment returns in majority of the countries in longer periods of time. Figure 1, which is obtained from Pensions Markets in Focus 2020 report plots the increase in the total amount of assets in retirement savings plans in the OECD countries and other selected countries in the 2009-2019 period.





Figure 1: Assets in Pension Funds in the OECD Countries and Other Selected Countries

It can be said that there are two main factors that affect the total pension fund size. First factor is the net cash flows to the system. If contribution payments made by the participants to

the pension system is higher than the benefit payments made by the system to the participants, then there are net positive flows to the system, and vice versa. Positive net flows increase the asset size in the private pension system, whereas negative net flows decrease the pension asset size under management. Second factor that affects the pension fund size is the changes in the financial markets, such as increases or decreases in stock exchange markets, money markets, precious metals, and currency rates. Portfolio managers aim to fine tune portfolio compositions in order to benefit from these changes in market prices.

This study aims to focus on this second factor that affects the total pension fund size. Main objective of this study is to utilize Python libraries in doing the required analysis and comparing results to suggest the best fitting model. This project must be regarded as an application of data analytics on Python more than as a study of applied econometrics. We will use total fund size data of one of the leading companies in the Turkish Private Pension Sector. This study exercises both univariate and multivariate analysis to predict the changes in total fund size in the upcoming 1-5 months, on average. Univariate analysis consists of naïve (persistence), autoregressive (AR), moving average (MA), and autoregressive moving average (ARMA) models. Multivariate analysis includes vector autoregression (VAR) and multiple linear regression (MLR) models.

The organization of this study is as follows: Chapter 2 presents a literature review about studies on pension systems with an emphasis on Turkish financial markets, model selection, and findings. Chapter 3 gives a brief explanation of the data with a discussion of stationarity of the series under study. Chapter 4 gives a brief discussion of models and the application of the models on data. Chapter 5 concludes with a summary of results. Python codes that are used in this project are presented in a separate Appendix document.

2. LITERATURE REVIEW

This study focuses on forecasting changes in the total fund size net of cash flows. Under the assumption of no cash inflow or outflows, changes in net asset value of the fund can be regarded as a proxy of changes in fund prices. Price change is a measure of return, thus a measure of performance. Therefore, regarding the pension funds we have narrowed our literature review to fund sizes, fund performances, and some applications in Turkey. Moreover, since we are focusing on univariate and multivariate models in time series forecasting, we have also focused on literature on uses of AR, MA, ARMA, autoregressive integrated moving averages (ARIMA), VAR, linear regression models, and use of artificial neural networks (ANN) in financial markets, with some applications in Turkey.

There are several studies about pension funds and fund performances in the literature. Del Guercio and Tkac (2002) compares pension funds and mutual in terms of fund manager and customer behaviors. Pension fund customers have long-term expectations and relatively tight benchmark performance comparison compared to mutual funds customers. Customers choose pension funds in accordance with their risk preference expect portfolio returns to reply benchmark returns. They punish the poor performers rapidly by shifting to different funds or pension companies. In response, pension fund managers are aware of customer expectations and benchmark sensitivity, thus they seek lower tracking errors and are more oriented in attaining benchmark returns for the portfolio compared to mutual funds managers [2].

Bikker (2015) discusses optimal fund sizes in terms of scale economies and lower operational costs versus fund performance. He argues that as long as there is unused economies of scale in the pension fund, operational efficiency will help increase fund performance as long as low cost investments are preferred, such as money market instruments and commodities over stocks and real estate investments. Changes in fund size is argued to occur largely due to some shocks like mergers and discontinuations of pension funds [3].

Ferson and Khang (2001) mentions the difficulties in measuring pension fund performances due to exogenous cash flows to the funds. They suggest an approach that uses portfolio weights with conditioning information that helps measure portfolio performance better despite the cash inflows to or outflows from the fund [4].

Andonov et.al (2012) discusses that although larger pension funds have the advantages of lower costs, they exhibit diseconomies of scale due to increased illiquidity. Larger funds may

perform better if they are not managed to replicate market movements but have a more passive stance instead [5].

In their 2007 article Bikker et.al. find out that regarding the performance of stock exchange markets, pension fund investment decisions made by the Dutch fund managers do not have a far sighted perspective. Investment behavior is driven by cyclical performance of the stock market rather than long term trends. Fund managers have a tendency to seek gains by market timing but are unable to correctly predict stock markets on average [6].

In their 2013 study Dağlı et.al examine Turkish Private Pension System in several parameters such as fund sizes, fund performances, fund standard deviations, brokerage commissions, proportions of securities in fund size, and fund ages. They make a panel data study for 29 funds and obtain some robust theory consisting findings in terms of correlations between fund performances and stock proportions and fund performances and brokerage commissions [7].

Korkmaz and Uygurtürk (2007) use single and multiple linear repression analysis to measure performances of 46 Turkish pension funds. They use Istanbul Stock Exchange 100 Index, KYD All GDS Bond Index and KYD O/N Net Repo Index as the independent variables and weekly percentage fund price changes as the dependent variable. Their findings suggest that all the variables are stationary. They come out with different findings for fund categories in terms of robustness and theory consistency [8]. Korkmaz and Uygurtürk (2008) study fund performances and portfolio managers' market timing capabilities using linear regression [9].

Wood and Dasgupta's 1996 study focuses on utilizing regression, ARIMA and ANN in predicting movements in the MSCI USA capital market index. They found out that root mean squared errors are lower with predicting through neural networks with one hidden layer compared to root mean squared errors of ARIMA and regression [10].

In their 1999 study Indro et.al. use an ANN approach to forecast the performance of equity mutual funds. They use a multi-layer perceptron model and a nonlinear optimizer assessing several independent variables such as fund turnovers, P/E and P/B ratios, market capitalization, and stock and cash ratios in order to predict fund performances. Their findings for growth funds (stock heavy funds) indicate superiority of ANN over linear regression models [11].

Zhang 2001 study uses a hybrid model of ARIMA and neural networks in time series forecasting. Author states the main disadvantage of the ARIMA model as its linearity assumption. ANNs, on the other hand, do not make such assumptions, and are more flexible models that can handle non-linearity. Real life time series examples are not purely linear or

non-linear, thus a combination of the two models is expected to perform better than sticking to one model. Zhang figures out that ANN performs slightly better than ARIMA in 1 month time span, yet the hybrid model performs better than the two. Model performances converge to each other in the longer timespans of 6 and 12 months [12].

In their 2012 study Kumar and Thenmozhi advanced Zhang's study with an addition of E-GARCH model to predict the daily closing prices of Nifty and S&P 500 Index. They have used a three-layer feed-forward ANN model to forecast daily returns of the indices. Results show that ANN model performs better than the linear ARIMA and E-GARCH models. Hybrid model that uses the residuals of the ARIMA and E-GARCH model in the ANN model performs better than the two, giving the lowest mean squared errors in predicting Nifty and S&P 500 returns [13].

Taskaya-Temizel and Casey (2005) findings contradicts Zhang 2001 findings to some extent. Authors showed that depending on the stochastic nature of the dataset under investigation, such trend, seasonality and non-linearity, usage of hybrid ANN models does not necessarily outperform linear models like ARIMA. In most of the cases hybrid models, as well as ANN models outperform linear ARIMA models, yet it is argued that there is no strict superiority of one model over the other regardless of the nature of the dataset [14].

There are several studies regarding applications of machine learning and deep learning on Istanbul Stock Exchange indices and stock prices. Gündüz et.al. (2017) forecasts the hourly BIST100 price changes by convolutional neural network (CNN) and logistic regression models, obtaining robust findings for their revised CNN model [15].

Aydın and Çavdar (2015) makes a comparative analysis of VAR and ANN in predicting BIST100 Index, gold prices, and USD/TRY exchange rate. Authors used a multilayered feedforward neural networks model and a VAR model on monthly data. They find out that an ANN model with 2 hidden layers performs better than the VAR model in forecasting of these variables [16].

Tekin and Çanakoğlu (2018) study uses linear regression, ANN, random forest and random tree algorithms to forecast selected stocks of İstanbul Stock Exchange. They find out that random forest algorithm provides better fit compared to other models [17]. Similarly Tekin and Çanakoğlu (2019) compares forecasting performance of ARIMA model, long short-term memory, linear regression, random forest, random forest algorithms, and KNN algorithms. They argue that ARIMA and linear regression have better fit compared to other models [18].

3. DATA

The aim of this study is to predict the changes in total pension fund size in the next month by utilizing different models and selecting the best fitting model for the dataset under investigation. In this study our main variable is the FUND variable. We have used the total pension fund size data of one of the leading pension companies in the sector, whose fund size amounts about %20 percent of the Turkish PPS on average for the October 2003 - November 2020 period, in other words from the beginning of the system to today.

Turkish private pension fund data are publicly available at the Capital Market Board [19] and Turkey Electronic Fund Trading Platform (TEFAS) [20] websites. Fund size, fund price, number of units in circulation, and net asset value data are publicly accessible on these platforms. We have used the data of all 34 funds of one selected pension company for simplicity since there are a total of 402 active funds in the sector as of November 2020. To note, not all of these 34 funds were existent since 2003, thus their asset values are simply zero before establishment.

Our FUND data is the daily net changes in total asset value of 34 funds excluding the net cash flows. Simply:

$$FUND_t = TNAV_t - TNAV_{t-1} - NCF_{t-1}$$

where; TNAV_t is the total asset value of the 34 funds at time t, TNAV_{t-1} is the total asset value of the 34 funds at time t-1, and NCF_{t-1} is the total net cash flows to 34 funds at time t-1. A change in total net asset values captures the daily growth (or decay) in total fund size. However, we want to get rid of the change in fund size due to additional cash to the funds or the cash payments made by the funds. Therefore we deduct the net cash flows (can be positive or negative) at time t-1 from the change in total fund size between time t and t-1. FUND now contains the information on changes in fund size due to changes in the market prices of the financial instruments contained in the funds. Any market factor that increases the prices of financial instruments in the funds leads to an increase in the FUND variable, and vice versa. Of course, all 34 funds in the size of one fund and a decrease in the other. For example, one fund can be in a long position for a specific instrument, suppose USD, where another one is on a short position, then an increase in USD/TRY exchange rate will have opposite effects on these two funds. However, we are interested in the final change for 34 funds, not in the fund specific

effects. Note that, FUND is in TL terms in the source file. After loading the data from the dataset in excel format (by use of Pandas), we have normalized FUND by subtracting the mean of sample from each observation and dividing each deduction by the standard error of the sample, known as z-score normalization.

In our univariate models only we use FUND series. In multivariate analysis FUND is the dependent variable and there are four independent variables; XU100, USD, XAU, and TRINT. XU100 is the İstanbul Stock Exchange 100 Index closing value, available on İstanbul Stock Exchange website [21]. Stock exchange variable is expected to affect FUND via the securities held by the funds. USD is the US Dollar to Turkish Lira spot exchange rate. USD/TRY exchange rate variable is expected to affect FUND via the foreign currency denominated assets held by the funds. We did not use an additional variable for EUR/TRY rate because of the multicollinearity of the two exchange rates. XAU is the dollar price of 1 ounce of gold. Gold price variable is in US Dollar terms so do not have a collinearity with our USD variable (USD/TRY exchange rate). It is expected to affect FUND via the physical gold and gold denominated assets held by the funds. TRINT is the closing value of interest rate on 1 year maturity Turkish governments bonds (for a few missing observations interest rates of 9 months or 6 months maturity government bonds are used). Interest rate on 1 year Turkish government bonds variable is expected to affect FUND via the by the funds.

We have preferred Thomson Reuters Eikon platform to obtain data for our independent variables due to our accessibility of the platform and its ease of use, yet these are public data that can be deducted from various public sources such as websites of banks, investment firms, investment websites, and the Central Bank. To sum up, complete dataset consists of daily observations of five variables; FUND, XU100, USD, XAU and TRINT, for the period of 28.10.2003 - 20.11.2020.

3.1 Testing for Stationarity

Non-stationarity of a time series implies presence of a unit root. Unit root indicates that current values of the series are persistently affected by its own past values, regardless of how many periods have passed. We need time series data to be stationary in order to use them in models like AR, MA or ARMA for forecasting. Time series are expected to be stationary as long as they have three features, constant mean over time (no strong trend), constant standard deviation over time (no major increase or decrease in volatility through time), and lack of seasonal patterns (no seasonality). Series with these three properties provide usability in univariate forecasting models mentioned above and we will also be safe to use them with other stationary variables in multivariate models like VAR. One can plot the data for a quick check of these three features for stationarity, however we need to use a formal test for stationarity in order to proceed on model selection. Of course, there are models that enable the use of nonstationarity series, like ARIMA in univariate analysis and ARDL in multivariate analysis for a mix of stationary and non-stationary series.

We will use Augmented Dickey-Fuller (ADF) test, which has the null hypothesis of the existence of a unit root. Existence of a unit root in the sample under investigation implies that the series is non-stationary, hence for stationarity we need to reject the null hypothesis. Note that, lower ADF test statistics compared to critical values implies rejection of the null.

Plot of FUND time series in Figure 2 shows constant mean and no seasonality, but we see higher standard deviation in the later years of the sample, yet ADF statistic value is lower than the critical value (ADF Test Statistic: -10.157678; critical values, %1: -3.43; %5: -2,86), p-value is lower than 0.05 (p-value: 0.00). We can conclude that FUND series is stationary (H₀: There is unit root. We reject H₀, series is stationary).



Figure 2: Plot of FUND series

FUND variable is our main interest in this study. In univariate analysis we will use models that undertake stationary variables for forecasting, such as AR, MA and ARMA. In case of non-stationarity we would have to use ARIMA, SARIMA, ARCH or GARCH to handle the unit root in the series.

Plot of XU100 time series in Figure 3 shows strong trend (increasing mean). ADF test statistic is higher than critical value (ADF Test Statistic: -0.756570; critical values, %1: -3.43;

%5: -2.86), p-value is higher than 0.05 (p-value: 0.831548). We can conclude that XU100 series is non-stationary (H₀: There is unit root. We do not reject H₀, series is non-stationary).



Figure 3: Plot of XU100 series

Plot of USD time series in Figure 4 shows strong trend (increasing mean). ADF test statistic is higher than critical value (ADF Test Statistic: 1.853157; critical values, %1: -3.43; %5: -2.86), p-value is higher than 0.05 (p-value: 0.998449). We can conclude that USD series is non-stationary.



Figure 4: Plot of USD series

Plot of XAU time series in Figure 5 shows non-constant mean over time. ADF test statistic is higher than the critical value (ADF Test Statistic: -1.029364; critical values, %1: - 3.43; %5: -2.86), p-value is higher than 0.05 (p-value: 0.742404). We can conclude that XAU series is non-stationary.



Figure 5: Plot of XAU series

Plot of TRINT time series in Figure 6 does not show persistent trend, but mean does not look constant through time. We must rely on our ADF test statistic, which is lower than %5 critical value (ADF Test Statistic: -3.060373, critical values; %1: -3.43; %5: -2.86), p-value is lower than 0.05 (p-value: 0.029628). We can conclude that TRINT series is stationary with %95 confidence.



3.2 Dealing with Non-Stationarity

In our multivariate analysis dependent variable will be FUND which is found to be stationary, yet three of the independent variables are found to be non-stationary. A use of stationary variables with non-stationary variables can be problematic in models like multiple regression or VAR. A mix of different ordered variables can be estimated in models like ARDL (short for autoregressive distributed lags). However ARDL is not a model that is suitable for forecasting, because it includes present values of the independent variables. Therefore we will stick to VAR and multiple linear regression as our multivariate models for forecasting.

A non-stationary series can be converted into stationary series with differencing, which is simply done by subtracting the first lag of the series from itself. Number of differences required to obtain stationarity is called the order of integration. We will take the first differences of XU100, USD, and XAU and run ADF tests for non-stationarity. If test results imply stationarity, will use the first differences instead of levels in multivariate analysis and conclude that levels are of order 1. If first differences are non-stationary, we will take the first differences of the first differences (same as second differences of the original series) and test for nonstationarity again (levels are of order 2 or higher).

Plot of XU100_D (first difference of XU100) series in Figure 7 shows constant mean. ADF test statistic is lower than the critical value (ADF Test Statistic: -17.968357; critical values, %1: -3.43; %5: -2.86), p-value is lower than 0.05 (p-value: 0.000000). We can conclude that XU100_D series is stationary.



Figure 7: Plot of XU100_D series

Plot of USD_D (first difference of USD) series in Figure 8 shows constant mean. ADF test statistic is lower than the critical value (ADF Test Statistic: -11.129131; critical values, %1: -3.43; %5: -2.86), p-value is lower than 0.05 (p-value: 0.000000). We can conclude that USD_D series is stationary.



Plot of XAU_D (first difference of XAU) series in Figure 9 shows constant mean. ADF test statistic is lower than the critical value (ADF Test Statistic: - 65.516171; critical values, %1: -3.43; %5: -2.86), p-value is lower than 0.05 (p-value: 0.000000). We can conclude that XAU_D series is stationary.



First differences of all the non-stationary variables are stationary. We can use XU100_D, USD_D, and XAU_D along with TRINT in our multivariate analysis.

4. MODEL

We will use univariate and multivariate models to forecast FUND data for one month ahead, which is for 20 observations since our data is given in working days. We will calculate mean squared errors of each prediction and compare our models accordingly. Univariate analysis includes naïve (persistence), AR, MA and ARMA models, multivariate analysis includes VAR and multiple linear regression.

4.1 Univariate Models

A series can only have explanatory power on itself as long as there is some level of correlation between the series and its lagged values, known as autocorrelation. If there is no autocorrelation between current and the past values of a series then the series is in "random walk" nature. In this case it is not possible to forecast future values of the series by using observed current and past values.

We will start with plotting lags and current values of FUND series. Lag plot of FUND in Figure 10 gives us an indication on the existence of autocorrelation. We can see that majority of observations are grouped together, implying existence of autocorrelation. Notice that y-axis is in $FUND_{t+1}$ values and x-axis is in $FUND_t$ values. We would see randomly distributed points on the plot if the series was a random walk.



Figure 10: Lag Plot of FUND

Lag plot implies that lags of series have explanatory power over current values. We can use autocorrelation graph of the FUND series as in Figure 11 to see how many lags of the variable have an effect on its current observation.



Figure 11: Autocorrelation Graph of FUND

For a visualization of 35 lags, we can see that plot of first 4 lags imply autocorrelation, since they are out of critical boundaries, lags from 5 to 8 are in the boundaries. We are safe to use 4 lags to start our models in order to reduce complexity. However, some higher lags are out of boundaries as well, we may have to take them into consideration.

4.1.1 Naive Model

Naive model (or the persistence model) forecasts the next values same as the last observation. Simply;

$$Y_t = Y_{t-1} + E_t$$

where, Y_t is the current value, Y_{t-1} is the immediate previous value, and \mathcal{E}_t is the error term. Note that model tries to fit the coefficient of Y_{t-1} to 1, model persistently predicts the next value same as the last one. This kind of forecasting (if we still can call it forecasting) can be useful only if the environment is prone to minimal change and any error in the forecast can be ignored. We can use such a model for the noon time temperature at a specific location in the tropics, or height of an orchid flower measured daily in the office. Naïve model is not useful for forecasting economic series.

Plot of test values (last 20 observations) of the sample and the predictions are given below in Figure 12. Notice that, predictions are the one period shifted version of the actual observations. Predictions copy the movement of the actual values one period behind. Mean squared error of the naïve model is found to be 22.51.



Figure 12: Plot of Naïve Model Predictions vs. Actual Data

4.1.2 Autoregressive Model (AR)

Autoregressive model predicts future values based on past values with a lower correlation suggested by the persistence model, which assumed past and present values are perfectly correlated. In AR model correlation between the past and the current values (autocorrelation) are less then unity, past values have an explanatory power over current values. Mathematical notation for an AR(p) process is as follows;

$$Y_{t} = c + a_{1} * Y_{t-1} + a_{2} * Y_{t-2} + \dots + a_{p} * Y_{t-p} + \mathcal{E}_{t}$$

where, Y_t is the current value of Y, c is the constant term, Y_{t-1} to Y_{t-p} are the past values of Y back to p periods, a_1 to a_p are coefficients of the respective past values of Y, and \mathcal{E}_t is the error term,.

Lag selection is very important in an AR model's predictive power. Additional lags increase model complexity but may improve the predictive power of the model. We can use partial autocorrelation graph of to pick for the AR lags or use statsmodels library's AR method

to pick for optimal lags. We will start with picking 31 lags suggested by the algorithm, which are greater than what partial autocorrelation graph suggests (4 lags).

Below in Figure 13 is the plot of AR(31) model predictions with actual observations. We can see that in AR model predictions fit better than the naïve model. Mean square error is 14.53, lower than the MSE of naive model (22.51). AR gives us a better fit.



Figure 13: Plot of AR(31) Predictions vs. Actual Data

In this model we are using 4264 past values to predict the 20 values. Deviations of the predictions from the actual 20 observations is captured in the MSE value. Note that we can use a time lapse method to make a prediction of the 20 values recursively, by predicting the first value same as before, with 4264 past values, and predicting the second value with the information in the 4264 past values and the first actual value of the test set. Increasing the number of observations in the past values (train set) by one additional value from the test set for which a prediction have been made in the last recursion, we end up predicting the 20th (last value) with the information of the 4283 actual observations of the sample. This kind of recursive prediction is called the walk forward prediction, which is expected to use maximum amount of information held by the sample and produce better fits as a result.

Following is the plot of AR(31) model with walk forward approach in Figure 14. Improvement in the model fit can be seen in the plot compared to the previous plot. MSE of the AR(31) model declined to 12.86 with recursion compared to with no recursion (14.53).



Figure 14: Plot of AR(31) Walk Forward Predictions vs. Actual Data

Use of 31 lags were suggested by the statsmodels' AR method. We have found out that AR(72) gives the minimum MSE (12.01) for a trial of up to 100 lags, plotted in Figure 15. We can improve our fit by employing walk forward approach to AR(72) model plotted in Figure 16, in which MSE declines to 10.04.



Figure 15: Plot of AR(72) Predictions vs. Actual Data



Figure 16: Plot of AR(72) Walk Forward Predictions vs. Actual Data

Since our data is daily, we cannot find an economic intuition in using such high number of lags. If our data were a high frequency data of seconds or milliseconds this many lags would imply a meaningful behavior of the series. We would suggest using up to 4 lags in AR model as suggested by partial autocorrelation graph in Figure 17. Note that lags up to 4 are significant. Lags between 5 and 8 are insignificant. 9th lag is significant. We can keep AR order at 4 to decrease model complexity in the ARMA section.



Figure 17: Partial Autocorrelation Plot of FUND

4.1.3 Moving Average Model (MA)

Moving-average is the model in which a series depends linearly on the current and the past errors which are the deviations the actual values of series from the expected value. MA(q) process is;

$$Y_t = c + \mathcal{E}_t + b_1 * \mathcal{E}_{t-1} + b_2 * \mathcal{E}_{t-2} + \dots + b_q * \mathcal{E}_{t-q}$$

where, Y_t is the current value of Y, c is the constant term, \mathcal{E}_i are the deviations (errors) of Y_i from the expected value for the periods from the current period back to q, and b_1 to b_q are coefficients of the respective past values' error terms.

Lag selection is critical in forecasting with MA model. MA model fails to effectively predict number of periods higher than the lags. An MA(q) model can make a good prediction of q periods ahead. After q periods MA predictions will converge to a mean and will remain unchanged. This problem can be handled with higher lag selection, or a recursive method (walk forward) as explained above in the AR section. We will use up to 20 lags to predict the 20 upcoming values of FUND variable by MA. We will also use a walk forward approach with lower order MA(q) models.

As partial autocorrelation plot gives an intuition of number of lags appropriate for using in AR, autocorrelation plot in Figure 18 provides an intuition for the number of lags to choose in MA. It looks reasonable to use 4 lags in MA, yet there are strong correlations for higher lags like 14. We can start with higher lags and narrow down our model later.



Figure 18: Autocorrelation Plot of FUND

We have first estimated MA(14) with the help of statsmodels library's ARIMA method setting p=0, d=0, and q = 14 (ARIMA(0,0,14) model is the same as MA(14)). Outputs show that, except for one lag, all the lags of the error terms are significant in explaining FUND variable. MSE of the MA(14) model is found to be 15.87, which is higher than the MSEs in the AR model, but lower than the naïve model. We can say that MA model definitely have some usability in FUND prediction. However, as discussed earlier, 14 lags are not enough to predict 20 periods ahead. As can be seen from the plots of MA(14) predictions and the actual data in Figure 19, last 6 predictions converged to a value and did not change at all.



Figure 19: Plot of MA(14) Predictions vs. Actual Data

To increase the predictive power of the MA model we can first increase the number of lags to the number of periods we want to forecast. MA(20) model estimation shows that, except for two lags, all the lags of the error terms are significant in explaining FUND. MSE declined to 15.53 compared to 15.87 of the MA(14) model. Prediction plot shows that all 20 predictions are unique as in Figure 20.

We can also improve the model power by adding recursion to the estimation as we have done in the AR section. An MA(4) model with walk forward approach gives MSE of 13.94 and a better fit of prediction in the plot. Walk forward approach definitely improved the prediction power of the MA model, can be seen in Figure 21.

We have found out that MA(10) with recursion, plotted in Figure 22, is the MSE minimizing MA model (13.71), and MA(4) with recursion is also very good compared to higher order MA models.



Figure 20: Plot of MA(20) Predictions vs. Actual Data



Figure 21: Plot of MA(4) Walk Forward Predictions vs. Actual Data



Figure 22: Plot of MA(10) Walk Forward Predictions vs. Actual Data

4.1.4 Autoregressive Moving Average Model (ARMA)

Autoregressive Moving-average model combines the autoregressive component of the series with its linear dependence on the current and the past errors. ARMA may improve fit of data compared to AR and MA. Model complexity increases, however we can decrease the number of total lags in ARMA compared to using AR and MA alone. ARMA(p,q) process is;

 $Y_t = c + \mathcal{E}_t + a_1 * Y_{t\text{-}1} + a_2 * Y_{t\text{-}2} + ... + a_p * Y_{t\text{-}p} + b_1 * \mathcal{E}_{t\text{-}1} + b_2 * \mathcal{E}_{t\text{-}2} + ... + b_q * \mathcal{E}_{t\text{-}q}$

where, Y_t is the current value of Y, c is the constant term, \mathcal{E}_t is the error term in current period, Y_{t-1} to Y_{t-p} are the past values of Y back to p periods, a₁ to a_p are coefficients of the respective past values of Y, \mathcal{E}_i are the deviations of Y_i from the expected value for the periods back to q, and b₁ to b_q are coefficients of the respective past values' error terms.

Lag selection is also important in ARMA(p,q) model. Since it combines the linear explanatory power of AR(p) and MA(q), to keep the model complexity limited, we can choose number of lags as proposed by the PACF and the ACF plots. Especially with daily data, a high number of lags do not have economic intuition as they do for higher frequency data of minutes and seconds. Our PACF plot suggested starting with p (order of AR) equal to 4, and ACF plot suggested starting with q (the order of MA) equal to 4 as well.

ARMA(4,4) outputs show that all 4 lags of AR and all 4 lags of MA terms are significant. We can say that ARMA(4,4) results are robust, yet MSE of the model is found to be 15.48 which is higher that our previous findings on AR and MA models, and the prediction plot in Figure 23 shows a poor fit of the model to data.



Figure 23: Plot of ARMA(4,4) Predictions vs. Actual Data

We have tried some other combinations of p and q in ARMA(p,q), with robust results in terms of lags' significance. Comparing ARMA (4,4) with ARMA(4,10), ARMA(9,4), ARMA(2,2), ARMA(1,3) we still see that ARMA (4,4) is the best fitting ARMA model to the data and gives the lowest MSE on the predictions versus actual test data.

We can improve the model fit with recursion. We see that walk forward approach improves the model fit in ARMA(4,4) model, as it did in AR and MA models. ARMA (4,4) MSE declined to 14.19 with recursion, yet with recursion the lowest MSE producing model is the ARMA(1,3) with an MSE of 14,15, so close to ARMA(4,4) with recursion. Figure 24 and Figure 25 show the plots of ARMA(4,4) and ARMA(1,3) with recursion.



Figure 24: Plot of ARMA(4,4) Walk Forward Predictions vs. Actual Data



Figure 25: Plot of ARMA(1,3) Walk Forward Predictions vs. Actual Data

4.2 Multivariate Models

A series can be explained by other series' current and lagged values, as well as its own lagged values. We will use vector autoregression (VAR) and multiple linear regression (MLR) models to forecast FUND data in this section. For sake of stationarity of the series, our explanatory variables will be the first differences of XU100, USD, and XAU variables, which are XU100_D, USD_D, and XAU_D and the levels of our TRINT variable, since TRINT is found to be stationary in the DATA chapter.

4.2.1 Vector Autoregression Model (VAR)

Vector autoregression model can be thought of as the multivariate extension of the AR model in which variables are explained by the lags of themselves and the other variables. All the variables under VAR are endogenous variables. If there are k many variables in the model all the variables will be collected in a vector Y_t of k*1 dimension in the left hand side of the model. A VAR(p) model would look like:

$$Y_t = c + A_1 * Y_{t-1} + A_2 * Y_{t-2} + \dots + A_p * Y_{t-p} + e_t$$

where, Y_t is a k*1 vector of endogenous variables values at time t, c is a k*1 vector of constants, A_i are k*k matrices of the coefficients, Y_{t-i} up to Y_{t-p} are the i-th lag of Y_t (so are k*1 vectors), and e_t is the k*1 vector of error terms.

Direction of predictability is very important in a VAR model. Mathematically, VAR method regresses all the variables onto others, yet we need a formal test of predictability, known as Granger Causality Test. This test can be applied to two variables, and shows us which variable "Granger causes" the other variable, in other words which can be used for the prediction of the other and which cannot. Some variables "Granger causes" each other, implying we can regress them on each other to make a prediction of the dependent one. Whereas, for some other couple of variables Granger causality works one way, we can explain one of them with the other, but not vice versa. We have conducted four Granger causality tests with FUND as the first variable and the other 4 variables as the second variable. All tests indicate that at least one lag of the XU100_D, USD_D, XAU_D, and TRINT variables are significant in predicting future values of FUND. So these 4 variables "Granger cause" FUND variable. We can use them for forecasting FUND variable in our VAR model.

In a VAR model, lag selection is important as it is in AR and MA models. For lag selection of each variable, first we look at the PACF plots of the series to determine maximum lag selection. Then we look at the Pearson correlations of lagged independent variables and our dependent variable FUND. PACF plots suggests using a low number of lags up to 3, and a

maximum of 48 lags for XU100_D, 4 lags to a maximum of 45 for USD_D, 5 lags and some other up to a maximum lags of 46 for XAU_D, and 2 lags to a maximum of 43 for TRINT. We used a maximum of 50 lags in our Pearson correlation tests between FUND and the independent variables XU100_D, USD_D, XAU_D, and TRINT for simplicity. We have found out that first 4 lags are significant for XU100_D (and some more up to 48), 2nd 4th and 5th for USD_D (and some more up to 48), first 4 lags for XAU_D (and some more up to 48, as well), but none for TRINT.

For the lag selection of the VAR model itself, we use two criteria, AIC (Akaike's Information Criterion) and BIC (Bayesian Information criterion). For a maximum of 100 lags, AIC suggests using 29 lags in VAR and BIC suggests using 2 lags. We have estimated both VAR(29) and VAR(2), and compared results.

We have used statsmodels library's VAR method in model estimation. VAR(29) has many insignificant lags for FUND as the dependent variable and an MSE of 12.49 for the predictions. VAR(2) model's all lags are significant for FUND as the dependent variable but the MSE increased to 16.02. We can use VAR(29) as our first multivariate model, plotted in Figure 26.



Figure 26: Plot of VAR(29) Predictions vs. Actual Data

Although AIC and BIC suggested using 29 and 2 lags respectively, we have increased the lag size up to 250 (a year in working days) and found out that MSE values kept decreasing as we increase our lag size. VAR model fit seemed to increase, such that MSE declined to 5.43

for VAR(72) (note that AR(72) is the MSE minimizing model in the univariate section), 4.66 for VAR(100) and to 0.72 for VAR(250). Figures 27, 28, and 29 indicate a better fit but we may suspect overfitting of the model to data as lag size is increased in VAR.



Figure 27: Plot of VAR(72) Predictions vs. Actual Data



Figure 28: Plot of VAR(100) Predictions vs. Actual Data



Figure 29: Plot of VAR(250) Predictions vs. Actual Data

4.2.2 Multiple Linear Regression Model (MLR)

Multiple linear regression model takes a dependent variable and regresses it to the given independent variables in order to fit the data on a line that minimizes the total deviations from that line. In simple linear regression there is only one independent variable. In multiple regression there are more than one independent variable, and also the lags of the independent variables can be used in the estimation. MLR can be used in forecasting as well as seeking long term relationships between variables, known as cointegration. However, there are more sophisticated models for cointegration like Johansen procedure and Vector Error Corrections models in the literature. In this study we are interested in forecasting rather than cointegration of FUND data, therefore MLR is our last model of interest in multivariate analysis.

An MLR model with levels of k independent variables would look like:

$$Y_t = c + b_1 * X 1_t + b_2 * X 2_t + \dots + b_k * X k_t + e_t$$

where, Y_t is the dependent variable time t, c is constant, $X1_t$ to Xk_t are the k independent variables at time t, and e_t is the error term. MLR is a very flexible model. We can add lags of the independent variables (for p as the number of lags; $X1_{t-1}$ to Xk_{t-1} and $X1_{t-p}$ to Xk_{t-p}), or we can omit the levels and only keep the lags (so no $X1_{t-1}$ to Xk_t terms) depending on the nature of our series and the model fit.

We have used XU100_D, USD_D, XAU_D, and TRINT as our independent variables to explain the FUND data in our MLR model. MLR results for regression between FUND and

the independent variables at time t, in other words when we use only the levels but not the lags of the independent variables, are not robust. R^2 value is found to be below 0, which is obviously an indication of poor fit of model. MSE is calculated to be 16.41, which is very high, and only better than the MSE of the naive model (which is 22.51) in the univariate analysis. Predictions of the MLR model with levels and no lags is plotted in Figure 30 below. We can see that predictions does not follow the actual data.



Figure 30: Plot of MLR Predictions without Lags vs. Actual Data

We have improved the MLR estimation by adding the lags of the independent variables to the regression. R^2 values increased to positive values, and MSE declined considerably. We have estimated the FUND variable on the levels of the independent variables and to their lags up to 20. We have found out that addition of first two lags to the levels as the explanatory variables provided the lowest MSE (9.71) and the highest R^2 (0.39). Results even improved when we have eliminated the levels from the model and estimated FUND on the first two lags of the independent variables, with MSE (9.49) and R^2 (0.40). Note that use of 2 lags is in line with the findings of BIC test results in our VAR section. Figures 31 and 32 plots the improvement in MLR predictions with the inclusion of lags to the model.



Figure 31: Plot of MLR Predictions with Levels and 2 Lags vs. Actual Data



Figure 32: Plot of MLR Predictions with 2 Lags and No Levels vs. Actual Data

CONCLUSION

In this study, we have utilized univariate and multivariate models to forecast the net changes in total pension fund size of a private pension company in Turkey, excluding the net cash flows to the funds. We have used daily data for the past 17 years. In the univariate section we have uses naïve, AR, MA, and ARMA models for forecasting, after showing the stationarity of the net changes in fund size series. In the multivariate section, we have used the VAR and MLR models to forecast net changes in fund size with four explanatory variables of stock market index, exchange rate, gold price, and interest rate.

Comparing the mean squared errors, we have found out that VAR models with high lags and MLR models with 2 lags of the independent variables outperformed the univariate models, as shown in Table 1. Best performing univariate model is the AR model with 72 lags for our data.

Model	MSE
VAR(250)	0.72
VAR(100)	4.66
VAR(72)	5.43
MLR, No Levels, 2 Lags	9.46
MLR, Levels, 2 Lags	9.71
AR(72) Walk Forward	10.04
AR(72)	12.01
VAR(29)	12.49
AR(31) Walk Forward	12.86
MA(10) Walk Forward	13.71
MA(4) Walk Forward	13.94
ARMA(1,3) Walk Forward	14.15
ARMA(4,4) Walk Forward	14.19
AR(31)	14.53
ARMA(4,4)	15.48
MA(20)	15.53
ARMA(4,10)	15.53
ARMA(9,4)	15.58
ARMA(1,3)	15.83
MA(14)	15.87
ARMA(2,2)	15.95
MLR, Levels, No Lags	16.41
Naive Model	22.51

Table 1: Mean Squared Errors of Model Predictions

We can summarize our findings as follows:

- i. Naïve (or persistence) model gives out the least accurate predictions, as compared over the mean squared errors between the predictions and the actual test data.
- ii. MA and ARMA fails to give good predictions as MA order is smaller than the number of periods we want to predict.
- iii. In AR, MA, and ARMA models increasing the order of p and q does not necessarily but usually improves the model fit.
- iv. Walk forward approach improves model fit on AR, MA, and ARMA models.
- v. VAR predictions improve as number of lags are increased. However we must be cautious about overfitting due to use of high number of lags.
- vi. Optimum MLR predictions are achieved with inclusion of 2 lags, and omission of levels of independent variables.
- vii. AR models with high orders gives good predictions. High lags would be more preferable with high frequency data, but they lack economic reasoning for daily data as in our study.
- viii. Multivariate models improved our predictions. Additional explanatory variables may help increase model fit.

Future studies can based on the last finding. Utilizing different or additional explanatory variable are likely to increase forecast success. We can suggest using a different interest rate variable such as KYD 365 GDS Bond Index (instead of our TRINT variable) and adding KYD Government Eurobond USD Index variable to the model to include the effect of changes in the Turkish Government Eurobond prices along with our independent variables.

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