

A Novel Standard for Graphical Representation of Mental Models and Processes in Cognitive Sciences

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Abstract

Cognitive Science has positioned itself to be a common ground in which models of mental processes from multiple disciplines merge, situating itself as a common field for new learning theories, or for formalizing existing ones. However, the authors have identified a need for updating the existing graphical representations by incorporating more accessible understanding for teachers and researchers in cross-multidisciplinary fields. In this regard, the present investigation attempts to generate a standard graphical language to represent complex mental processes by the introduction of functional principles, schemes and models that have been successfully used in technical areas such as adaptive control systems, algorithm flow charts, and artificial intelligence. This graphical representation, entitled “Cognitive Functional Representation” (CFR), is further shown to be efficacious in incorporating the essence of complex cognitive theories.

1. Introduction

Cognitive Science has addressed modelling and validating theories in sciences ranging from Biology to Neuroscience, and from Psychology and Artificial Intelligence [1]. Absorbing and reviewing these theories in different contexts, Cognitive Science has positioned itself as a common ground in which models of mental processes from multiple disciplines merge. However, the authors have identified a need for more complete, graphic representations that incorporate developments in the field, and potentially make these theories more widely accessible for teachers and researchers. Since the first proposition of mental representation, such as Johnson-Laird’s reasoning models [2], most advances have been focused on the topic of knowledge representation in the fields of artificial intelligence and cognitive informatics [3].

While graphical representation of mental models and their processes are needed for a better understanding of the different theories, the analysis,

evaluation and integration of the models usually do not deal with actual teaching purposes across multidisciplinary fields. In this regard, this investigation attempts to generate a standard graphical language to represent complex mental processes by the introduction of functional principles, schemes and models, which have previously been successfully used in technical areas such as adaptive control systems, algorithm flow diagrams, or artificial intelligence. This graphical representation, named by the authors as “Cognitive Functional Representation” (CFR), is further shown to be efficacious in incorporating aspects of complex theories such as Jean Piaget’s Theory of Intellectual Development.

2. Types of cognitive representation models

To understand how generic cognitive models can be graphically represented, one must understand the types of possible representations describing cognitive processes.

Cognitive models and processes are formed by static components such as concepts, facts, etc., and dynamic components like skills, habits, procedures, intentions, actions, speech, stimuli, attention, sensation, perception, memories, awareness, emotions, feeling, behavior, experiences, tasks, thoughts, reasoning, ideas, beliefs, values, attitudes, instruction, scaffolding, insight, etc.

In Thomas Hobbes’ Representational Theory of the Mind (RTM), he stated that knowledge is the evidence of truth, and which must have four properties [4]: first, knowledge must be integrated by concepts; second, each concept can be identified by a name; third, names can be used to create propositions; and fourth, such propositions must be concluding. RTM was the first formalization of this philosophy [5], and Jerry Fodors’ Language of Thought Hypothesis is one its latest extensions, stating that thoughts are represented by a language supported by the principles of symbolic logic [6].

In Osherson and Smith's "Classic Theory of Concept Representation" [7], the theory defines concepts as the representation of a mental object and a set of attributes, expressed through a specific language of the mind by symbols or patterns, but also considering descriptive capabilities, in the same manner as in Murphy and Medin's "Concepts as Theory Dependent" [8].

In Psychology, some of the theories to understand and interpret mental processes are the associative theories, also referred to as connectionist theories, the cognitive theories, and the constructivist theories.

Behaviorists do not consider internal cognitive processes, but only external behaviors to different stimulus, and for this reason, behaviorist theories cannot explain thought in desired depth [9].

Connectionist theories state that knowledge can be described as a number of interconnected concepts, with each concept connected through associations. These are the roots of semantics as the means for knowledge representation [10]. Semantic knowledge and similarity representation have been proven to be drivers of reasoning for unstructured knowledge [11,12]. Traditional connectionist approaches do not account for causality, but they focus on the presence or lack of associations and their amount, suggesting that these theories may not explain higher cognitive processes.

Constructivist theories consider more complex reasoning components, such as causality, probability and context. In this approach, each group of associations integrates different layers of thought differentiated by the strength of their associations. In the highest layer are found the concepts, and in the lowest layers, the ideas [10]. This structure leads to representations of complex mechanisms.

Brown [13] states that knowledge is composed of theories, causal explanation, meaningful solutions and arbitrary solutions. Theories are networks of concepts, causal explanations are facts, meaningful solutions are isolated pieces of knowledge and arbitrary solutions are random decisions.

Other approaches include Doignon and Falmagne's Knowledge Space Theory [14], which defines knowledge as a group of questions, which are combined with possible answers to form knowledge states, where their combinations create a congruent framework for knowledge.

The common components in all these theories for knowledge and thought representation consider the following [3]:

1. Knowledge is composed of concepts, which have attributes and network structures.

2. Concepts have associations with other concepts, and the associations have characteristics such as type, directionality, name, intension, extension, among others.

3. Associations and concepts lead to dynamic structures, which tend to become stable through time, becoming factual knowledge.

Concepts, associations and their structures apply for both factual (declarative) knowledge, and process (procedural) knowledge, although they are more natural to see in factual knowledge, since in processes, concepts are integrated and usually referred to as skills and competencies, and their relations are associated with rule sets. Structured knowledge strongly relies on information analysis using higher cognitive processes. Unstructured knowledge relies on lower cognitive processes such as associative knowledge and similarity [12,15,16]. Unstructured knowledge becomes structured when higher cognitive processes such as acquisition, ordering taxonomy, domain, direction of causality, and associations, among others, are applied. Even though some computer systems may undertake this process with semantic networks and Bayesian causality networks, they only accomplish it on an intuitive basis [12]. These, then, are the main building blocks of cognitive representations.

Knowledge representation is deeply linked to learning and reasoning processes, for example, Crisp-Bright [12] defines knowledge as "the psychological result of perception, learning and reasoning". Following this definition, in order to have any higher-level cognitive process, knowledge must be generated, represented, and stored. Both Newell's Unified Theories of Cognition [17] and Anderson's Adaptive Character of Thought [18], have strongly influenced today's cognitive representation models. In the Cognitive Informatics Theoretical Framework [19], the authors state "computerized knowledge representations are required in order to develop computerized systems with cognitive capabilities" [20].

Metacognition, a key cognitive process meaning cognition about cognition [21], has two fundamental aspects: metacognitive knowledge and metacognitive activity. Metacognitive knowledge involves monitoring and reflecting on one's current thoughts, which includes factual knowledge, such as knowledge about the task, and one's goals, and strategic knowledge, such as how and when consciously manage specific procedures to solve problems [22].

The most important types of cognitive representation models are symbolic, non-symbolic, declarative, and distributed (neural or probabilistic) networks, among others [3]. They differ in their suitability for representing types of reasoning, such as inductive, deductive, analogy, abduction, etc. [23].

Symbolic representation models include structures such as semantic networks, rule-based systems, frames, scripts and ontology-based concept maps.

Semantic networks are basically concept networks, where the concepts are represented as nodes, and associations as arcs [24], and can be defined as a graphical equivalent for propositional logic [25]. This type of representation model strongly relies on similarity, contrast and closeness for conceptual representation or interpretation. In semantic networks, associations have a grade that represents the strength of the association. In this way, learning is presented by increasing the association grade, or creating new associations between concepts.

Semantic networks are based on the traditional Representational Theory of the Mind and associative theories. They are mainly used to model declarative knowledge both in structured and unstructured ways, but they also can be used for procedural knowledge. When modelling structured knowledge, the associations are directed, including causality and hierarchy. Traditional semantic networks only used presence or absence of associations, however semantic networks such as MultiNet or Object Attribute Relation [26] provide deeper types of associations and integrate layers for knowledge composition.

Ruled-based systems are representation models focusing mainly on procedural knowledge, but they may be also used for classification purposes in declarative knowledge. They are organized as libraries of rules in the form of condition–action, or if–then–else sentences. Rule systems are excellent in representing skills, learning and problem solving [27,18].

Frames are data-structures representing stereotyped situations [28]. Frames are a kind of semantic network, combining declarative and structured procedural knowledge. Frames differ from networks in that they can include procedures. Every frame symbol contains a procedure (demon) [28], and a group of attributes for the situation description. The purpose of the frame is to emulate the human memory, which stores situations, combining procedural and declarative knowledge. This approach is an attempt to unify different approaches in psychology, linguistics and artificial intelligence.

Scripts are similar to frames. In Schank's theory of Scripts [29], scripts are sentences describing an action. Script theory was originally addressed to understand human language with episodic memory. Scripts are descriptions of a larger plan, which can also be used to model networks similar to those of semantic networks. Schank later used scripts in his Dynamic Memory Model [30] to explain higher aspects of cognition.

Neural networks are the most important types of distributed representation models, which use symbols to represent concepts, as well as neuron-like activation patterns to identify concepts or ideas. Neural networks emulate the cognitive process of

mental reconstruction of general idea patterns even if specific concept parts are lost in the process, and strengthening the patterns every time the brain thinks (referring this as training of the network). Both symbolic and distributed systems use concepts as units of knowledge. The difference is that symbolic models represent concepts as symbols, and the distributed ones as patterns. Both approaches use associations between concepts, and their association configuration as knowledge representation.

Ontology-based concept maps are explicit specifications of conceptualizations, or abstract, simplified views of the world [31]. Ontologies are flexible hierarchical structures in first order logic, defining relations between elements. Ontologies are agreements in social contexts to accomplish guiding objectives [32]. Ontologies also apply to folksonomies [33]. Their implementations resemble taxonomies or concept networks [34], i.e., semantic networks with formal conceptual descriptions for their associations, and hence, can be considered as symbolic systems. Ontologies may also include additional layers for the representation of concept embodiment.

4. Examples of representation models

The best representation models from the qualitative and quantitative point of view are found in the field of Artificial Intelligence, which are also known as knowledge representation models. In these models, associations are vital to knowledge representation in these representation models [5,6,10,35,36], the differences are in their properties and implications.

Most factual knowledge representations use propositional logic or its graphical equivalents in network representations. The specific type of network is determined by its association directionality [37], association type [38], inclusion of association cycles, hierarchies, grouping schemes, and their combinations.

The capacity to represent multiple contexts in a single representative example or instantiation, and the impact that context changes have on a concept's meaning are important representation functionalities.

The Micro-theories models used in a “Cyc” commonsense knowledge base [39] contemplate these features, combining multiple facts of a subjective nature into a coherent knowledge base. However, Cyc has proven to be a problem for most users [40], since it requires a specialized language based on predicate logic semantics for information modelling and extraction.

Graphical-oriented models such as MultiNet [37] or Object Attribute Relation (OAR) [26] have been used for natural language processing, knowledge composition, and process specification, however, their focus is not to represent several contexts at a

time. MultiNet has specific context differentiators based on grammar attributes, but not for concept meaning in changing contexts. Only a small fraction of concept information is usually reused, and the majority has to be rewritten for each domain. OAR shows a similar situation since the context is defined as a relation between objects and their attributes [26], but OAR is more flexible, and includes multiple contexts, however, not for the concepts. The concepts themselves are not dynamic, but the context objects. In consequence, a concept can have several different instantiations depending on the context.

The Memory Map (MM) model is a knowledge model that represents the interaction of concepts and skills in different contexts, including concepts with changing meaning according to contexts [3]. MM models are directed graphs similar to semantic networks and ontologies but with context flexibility; open granularity subject to modeler's criteria; arbitrary level of atomicity for each concept; and dynamic hierarchies changing for each domain. This type of representation model deserves a more detailed explanation, since it is the closest model to the one proposed by the authors.

The MM model is composed of concept representation units (CRU) represented as round nodes; Skill Representation Units (SRU) represented as square or rectangular nodes; and Associations among them represented as arcs. The concept and skill attributes together with their associations define the semantics and the corresponding knowledge. Concept meaning changes depending on the attributes tagged to corresponding domains. Concepts and skills have levels of knowledge, which are thresholds indicating the structure strength (weak syncretic or strong and stable, conceptual). Unlike concepts, only skills' associations have an application-oriented nature. Associations also include a level of strength. Attributes are a combination between concepts and associations. There is no distinction between objects and instantiations. The context (embodiment of semantic knowledge) is composed of one or several domains. Associations must belong to at least one domain, and by the combination of multiple domains, different contexts are formed.

Knowledge extraction is done by simple unguided recursive searches that return relevant model segments. Model's focus is to easily access information for open queries such as: knowledge about certain concept or skill, concept's attributes, relation of concepts under a particular domain, etc. New knowledge is acquired by associating it to existing knowledge.

The MM model has the following properties:

- 1) There is an unlimited number of levels (granularity).
- 2) Concepts and skills can be integrated into hierarchies through roles and directionalities. The

combination of hierarchical domains generates new dynamic context-dependent hierarchies (taxonomies).

3) The network is developed succinctly, avoiding redundancy, and delimited by contexts.

4) There is no limit for the number of units, hierarchy of attributes and associations, or knowledge depth in a structure.

5) The structure is flexible, creating associations between any unit, and any association can have several roles each offering a unique behavior.

The model has some restrictions:

1) A domain may appear isolated (secluded knowledge), however this is because an association may link certain concept or skill to a structure of a different domain that cannot be seen.

2) Associations must always be linked to units.

3) New concept or skill representation units must be created first, then their associations.

An example of a detailed MM Model for an advanced learning environment can be consulted in Ramirez and Valdes' "General Knowledge Representation Model" [3]. The model adapts user profiles, containing user knowledge, interest, learning styles and emotional profiles.

Certain similarities of knowledge representation models can be seen in Carchiolo *et al.* [41], and Van Marcke's approaches [42].

Specific knowledge representation models applied to education are the Intelligent Computer Aided Instruction ICAI [43], Adaptive Hypermedia [44], and the Intelligent Tutoring System ITS [43].

2. The Cognitive Functional Representation (CFR) Model

The Cognitive Functional Representation (CFR) Model is a symbolic representation model suitable for representing complex cognitive concepts and processes with the following characteristics:

- Directed associations with hierarchies, causality, and sequential logic.
- Explicit input/output information, comparisons, feedback loops, and disturbance inputs affecting the cognitive process.
- Functional blocks, their attributes representing concepts, skills, experiences, contexts, and processes, as well as open granularity subject to criteria and concept atomicity.
- Special focus on effective Learning Loops or association cycles emulating the cognitive process of mental reconstruction of knowledge (constructivism).
- Capability to represent not only factual or declarative knowledge, but easily represent dynamic structures, strategic and procedural knowledge, and consequently, skills and competencies related to rule-based sets.

- Representation of both factual and procedural knowledge of mainly structured knowledge of higher cognitive processes (acquisition, taxonomy, domain, direction causality, and association), but also unstructured knowledge of lower cognitive processes (such as associativity and similarity).
- Graphical representation of association strength and potentiality to represent complex network structures and cognitive representation processes such as inductive, deductive, analogy and abduction reasoning.

Each of these aspects represents novel graphical representation standards that are depicted in the following section with an example.

Besides the unifying approaches in psychology, linguistics and artificial intelligence, CFR also integrates the approaches of Adaptive Control Systems.

The CFR does not include propositional logic, and quantitative aspects such as association strengths, and activation patterns at this stage, but it may be further developed in the future.

3. Explaining Jean Piaget's Theory of Intellectual Growth in One Minute

The title of this section is to raise awareness about the power of CFR modeling in explaining complex theories such as Jean Piaget's Theory of Intellectual Growth [45-49], which was selected as a constructivist jewel, flagship of the cognitive sciences. This model is shown in the Annex at the end of the paper.

As it can be seen, a classical Adaptive Control System seems very suitable to represent Jean Piaget's theory. Besides meeting the traditional properties of the semantic networks and advanced models such as the Memory Map Model, the CFR model introduces new properties to the model:

- "Feedback Loops" that acquire the perceptions from reality by sensing stimuli, events and experiences, but also after the preliminary Assimilation of the reality.
- "Comparison Function" between the recent acquisition of the reality with the knowledge of the existing mental schemes.
- "Learning Actions", such as the association arrow coming from "Knowledge Construction" to "Accommodation" by the modification of the existing schemes to account for the new experiences, and promoting intellectual growth. The "Learning Action" arrow is usually represented by diagonally crossing through the block that is to be enriched or modified. Learning actions are also common in neural networks and deep learning algorithms used in artificial intelligence. The application of CFR

suggests that a "Learning Action" arrow is missing for the "Assimilation" block.

- "Input signals", which, by the way, are not provided in Piaget's theory, but that can easily result as consequence of using such adaptive control system, where for example, the signals may be seen as "inquiries", "goals", "orders", etc.
- Regarding output signals, any outcome arrow leaving from a block unit can be considered as an output signal in control systems. In the model presented in the Annex, the output is the "Equilibrium of Cognitive Structures".

Besides, other elements are added to enrich the proposed model:

- "Influential Factor" associations represented as "lightning" symbols are meant to affect a process by the direct or indirect action of other factors. A number of lightning symbols can be used to represent the level of influence or "strength" of the association in the process in a more visual fashion.
- "Clouds" are meant to represent factors that may directly or indirectly affect a process, such as the "External Environment". The CFR model also helps to identify deficiencies in the represented model, since it may be suggested that an additional "cloud" may be missing above the "Existing Model" that may include "Beliefs", "Values" and "Memories", among others, that drive or may affect the existing model.

There are other CFR elements that are not included in the application example due to its specific nature, but that may be used in other more procedural theories, and such elements are "Milestones" and "Hierarchies".

3. Conclusion

After a summary of the main types of representation models used for cognitive processes, a description of main specific examples of representation models and their properties, a novel graphical representation standard has been introduced and applied to the representation of a complex cognitive theory, such as Jean Piaget's Theory of Intellectual Growth.

The application of CFR to the concrete case of Piaget's theory unveils novel properties such as: "Feedback Loops" acquiring perceptions from reality; "Comparison Function" between existing and actual schemes; "Learning Action" association arrows that modify existing schemes and that directly account for the new experiences and intellectual growth; "Input signals"; "Influential Factor" associations directly or indirectly affecting a process and introducing "strength" of the association; "Clouds" representing factors directly or indirectly affecting a process; "Milestones" and "Hierarchies".

The application of the CFR model to Piaget's theory unveiled as well the unexpected outcome demonstrating that CFR can be used to detect deficiencies of original represented models while they are mapped into an adaptive control system, such as a missing "cloud" above the "Existing Model" that might include "Beliefs", "Values" and "Memories", among others, that drive or affect the existing model; inputs to the theory; and lack of a "Learning Action" for the "Assimilation" block.

At this stage, the CFR includes mainly qualitative properties and not propositional logic, and quantitative aspects such as quantitative association strengths, and activation patterns, but this may be integrated in further future developments.

4. References

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Annex: Cognitive Functional Representation Model of Jean Piaget’s Theory of Intellectual Growth.

