Bilevel Models on the Competitive Facility Location Problem

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Abstract Facility location and allocation problems have been a major area of research for decades, which has led to a vast and still growing literature. Although there are many variants of these problems, there exist two common features: finding the best locations for one or more facilities and allocating demand points to these facilities. A considerable number of studies assume a monopolistic viewpoint and formulate a mathematical model to optimize an objective function of a single decision maker. In contrast, competitive facility location (CFL) problem is based on the premise that there exist competition in the market among different firms. When one of the competing firms acts as the leader and the other firm, called the follower, reacts to the decision of the leader, a sequential-entry CFL problem is obtained, which gives rise to a Stackelberg type of game between two players. A successful and widely applied framework to formulate this type of CFL problems is bilevel programming (BP). In this chapter, the literature on BP models for CFL problems is reviewed, existing works are categorized with respect to defined criteria, and information is provided for each work.

1 Introduction

Facility location problems (FLPs) that arise as real-life applications in both public and private sector try to determine the optimal location for facilities such as warehouses, plants, distribution centers, shopping malls, hospitals, and post offices. They can have different objectives such as maximization of the profit obtained from customers and minimization of the costs incurred by locating facilities and serving customers. The basic FLPs are given in Daskin [14] as *p*-median, set covering, max-

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imal covering, fixed-charge, and hub location problems. A p-median model tries to minimize the demand weighted total or average distance between the customers and their nearest facility by locating p of facilities. A set covering model, on the other hand, attempts to minimize the number of facilities to be opened necessary to cover all demand points. In contrast to a set covering model, a maximal covering model assumes that it may not be possible to cover all the demand points by facilities. Hence, it locates a fixed number of facilities to cover most of the demand. All the models mentioned so far are uncapacitated facility location models and neglect the transportation costs between the customers and facilities as well as fixed costs of opening facilities. In contrast to these models, fixed-charge models take into account a limited capacity for facilities and transportation costs to serve customers using functions of distances and also fixed costs for locating new facilities. Hence, the total cost which is comprised of fixed cost and transportation cost needs to be minimized in order to determine the optimal number and locations of facilities, and the allocation of demand points to the opened facilities. Unlike the other models, a hub location model includes both the location of facilities referred to as hubs, and the design of the network by determining the hub node that is going to be assigned the every non-hub node.

Many factors influence the location decision for a new facility in a market, but one of the most important factors is related to the existing facilities which belong to competitors offering the same or similar commodities or services. When there is no competitor in the market, the new facility will be the only supplier of the commodity or the service leading to a monopoly in the market. However, if there are already existing facilities in the market, then the new facility will have to compete for customers with the aim of maximizing the market share or the profit (see Drezner [17]). Even the new facilities that are a monopoly at the market entry may face competition later when other competitors enter the market. In this chapter, we consider FLPs in a competitive environment, namely Competitive Facility Location (CFL) problems. These problems are spatial interaction models where a firm or franchise wishes to locate new facilities in a market with already existing or prospective competitors. In some problems, the firm or franchise may have existing facilities with known locations and attractiveness levels in the market, while in others the firm may be a new entrant with no existing facility.

Several CFL models are proposed in the literature so far, see for instance the survey papers by Eiselt et al. [21], Eiselt and Laporte [22], Plastria [38], Drezner [18], Eiselt et al. [23], and the references therein. These survey papers group the studies existing in the literature according to different model components. In fact, the most important factors that can be used to differentiate studies on the CFL problem are twofold: existence of the follower's reaction as well as the timing of action and reaction of the players in the game. CFL models with static competition ignore the reaction of a firm to the opening new facilities or redesigning existing facilities by other competitors. CFL models with foresight, on the other hand, take this reaction into account, and hence are more difficult in general compared to static models. The timing of reaction divides the CFL problems with foresight into two major classes, namely simultaneous-entry CFL problems and sequential-entry CFL models. In simultaneous-entry CFL problems, where a Nash game is involved, firms or franchises simultaneously make their decisions on the facility locations and other design components, if any. In contrast, there exists a precedence of decision making among the competing firms in sequential-entry CFL problems. These problems are generally recognized as a Stackelberg type of game between two firms (see von Stackelberg [48]). These games consist of a new entrant firm or a firm with existing facilities that establishes new facilities in the market, where an existing or a future competitor is present to react to the action of the first firm. This action-reaction situation brings us to the so-called two-level or bilevel programming (BP) problems, which constitutes the backbone of the chapter. BP problems include two independent players, namely the leader and the follower, who act sequentially with the objective of optimizing their own objective functions. The leader selects a strategy to optimize its objective function with the foresight or anticipation that the follower reacts to the chosen strategy with the aim of optimizing its own objective function. Furthermore, these objective functions are usually in conflict with each other as indicated by Moore and Bard [36]. The purpose of the chapter is to provide a review on the recent studies that include bilevel models within the context of CFL problems.

An example of a general BP problem is depicted in Fig. 1. In this example, both the leader and the follower own existing facilities in the market with known locations and attractiveness levels, and customers are aggregated at demand points as shown by small circles. First the leader, then the follower can install new facilities at candidate facility sites.

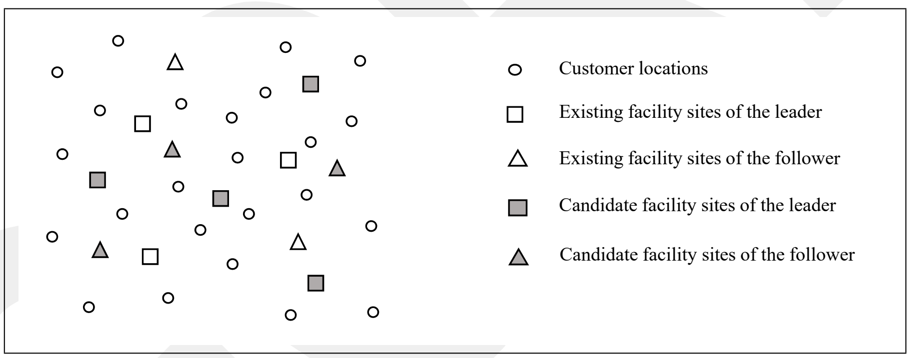

Fig. 1 An example of a CFL problem in discrete space

Since papers including BP models are reviewed in this chapter, the reader may benefit from visualizing the overall structure of a BP model. By letting $\mathbf{u}$ and $\mathbf{v}$ denote the decision variables of the leader and the follower, respectively, we can define a BP model as follows:

$$\begin{aligned}
\max_{\mathbf{u}} \quad & F(\mathbf{u},\mathbf{v}) \\
\text{s.t.} \quad & G(\mathbf{u},\mathbf{v}) \leq 0 \\
& \max_{\mathbf{v}} \quad f(\mathbf{u},\mathbf{v}) \\
& \quad \text{s.t.} \quad g(\mathbf{u},\mathbf{v}) \leq 0.
\end{aligned}$$

Here, $G(\mathbf{u},\mathbf{v})$ and $g(\mathbf{u},\mathbf{v})$ are the constraints of the leader and follower, respectively, while $F(\mathbf{u},\mathbf{v})$ and $f(\mathbf{u},\mathbf{v})$ are their objective functions. Integrality restrictions may exist on the decision variables **u** and **v**. We have to emphasize that such a BP model is formulated from the viewpoint of the leader. It can be easily seen that the so-called upper-level problem (ULP) of the leader incorporates the optimization of the lower-level problem (LLP) of the follower as constraints.

All CFL models attempt to infer the market share captured by each facility. Nevertheless, the captured market share can only be computed when we know the patronizing behavior of customers, i.e. the rules to allocate the customers to facilities. In most of the CFL models, the customers choose the facilities solely based on their proximity to the facility locations or on their preferences towards the facilities. On the other hand, there also exist CFL models that account for the attractiveness level, i.e. the design or the quality of facilities. The first CFL problem presented by Hotelling [27] considers a model involving two identical ice-cream vendors along a beach strip, namely in a linear market, where the customers are attracted to the closest vendor. This kind of facility location games is then developed by Hakimi [25] who also established the first fundamental complexity results. He further introduced two terms into the location science terminology, namely the $(r|p)$-*centroid problem* and the $(r|X_p)$-*medianoid problem* for the leader's and follower's problem, respectively. The $(r|X_p)$-medianoid, or the medianoid problem in short, refers to the follower's problem which tries to optimally locate r new facilities when the locations X_p of leader's p facilities are known. On the other hand, the $(r|p)$-centroid problem solves the leader's problem by opening p new facilities with the anticipation that the follower will locate r facilities in return by solving the $(r|X_p)$-medianoid problem. The CFL literature actually started with identical facilities, but expanded later with models that consider unequally attractive facilities.

Hence, the CFL models can be further divided into two classes by the customer choice rule: deterministic utility models and random utility models. In both classes, customers visit the facilities according to a utility function which is composed of the facility characteristics and the distance between the customers and the facility sites. In other words, if a facility has $n-1$ attributes such as a_1, a_2, ..., a_{n-1}, then the utility T of this facility is defined as $T = T(a_1, a_2, \ldots, a_n)$, where a_n represents the distance between the customer and the facility as the facility's nth attribute. The major distinction between the deterministic and random utility models is that in deterministic utility models customers choose the facility that provides the highest utility to them, which implies the all-or-nothing property. In other words, a customer visits a facility with a probability of either zero or one. Drezner [17] points out that the utility function T is usually an additive function such that $T = \sum_{i=1}^{n} \beta_i z_i(a_i)$ where β_i is the weight of the ith attribute and f_i is a function of a_i. If the utility function in a deterministic model is additive, then most of the time the notion of *break-even distance* is used. This is the distance at which utilities of new and existing facilities are equal. In such a case, customers patronize a facility if and only if the facility is opened within the break-even distance. On the contrary, the utility function in a random utility model varies among customers and each customer draws his/her utility from a random distribution of this utility function. Hence, the probability that a customer visits a facility varies between zero and one. Random utility models are basically discrete choice models for short-term travel decisions. A discrete choice model can be employed when an individual decision maker faces a finite number of alternatives, where each alternative is characterized by a set of attributes. In case of a CFL problem, the decision makers are the customers aggregated at demand points, alternatives are the facilities, and attributes are various facility characteristics. The last component that describes a discrete choice model consists of the decision rules according to which the decision makers make a choice among the alternatives (see Ben-Akiva and Bierlaire [4]). The most widely used random utility model in CFL literature is the gravity-based model which was first proposed by Reilly [44] and then employed by Huff [28, 29]. It assumes that the probability that a customer visits a facility is proportional to the attractiveness level of the facility and inversely proportional to a function of the distance between the customer and the facility. The attractiveness level, which can also be names as the design or the quality of the facility, is composed of various facility attributes such as the size or the floor area of the facility, the variety of the products sold, the prices offered by the facility, the existence of a parking lot, and the proximity to public transportation.

We first provide a classification of the studies that are relevant within the scope of this chapter, namely works that develop a BP model for the CFL problem. The second and third columns of Table 1 categorize the studies according to leader's action and follower's reaction. The main objective of any FLP is to find the optimal location for new facilities. Thus, in every CFL study except Küçükaydın et al. [32], both players wish to determine the optimal location ("L") of their new facilities. However, a relatively few number of papers also consider the design ("D") of new or existing facilities as a decision variable. Thus, it is possible that the ULPs and LLPs include only the location decisions ("L") or only the design decisions ("D") or both ("L+D"). In Küçükaydın et al. [32] the follower's reaction only consists of design decisions of the existing facilities. Furthermore, Fischer [24] determines the optimal facility location as well as product prices offered by the facilities for both players which can be recognized as a ("L+D") decision.

It is possible to categorize the CFL problems (as is the case with all FLPs) with regard to the set of candidate sites for opening facilities, which can be a discrete set, a continuous set, or a network. Thus, the next column of Table 1 differentiates the studies according to this criterion. Discrete CFL problems consider predetermined facility sites as candidate locations for the new facilities, whereas in continuous CFL problems it is possible to locate a facility anywhere in the plane. Finally, in network-based problems, the candidate sites are the nodes and edges of the network. All but one study existing in the table are identified as either a discrete ("Disc") or a continuous problem ("Cont"). The only study that involves a CFL problem defined on a network is due to Tóth and Kovács [49]. Thus, it is classified as a network problem ("Netw").

The customer choice rules that are found in the papers reviewed are as follows: "Distance (Dist)", "Preference (Pref)", "Price (Price)", "Gravity-based (Gravity)", and "Proportional (Prop)" in the fifth and sixth columns of Table 1. The first three of these rules can be thought of as a deterministic customer allocation, whereas the last two can be seen as a probabilistic allocation. The papers classified in the deterministic choice rule group allocate the customers either to the closest facility, or to the most preferred facility, or to the facility offering the lowest price. In the case preference is used, each customer ranks the facilities according to a "linear" order. The papers in this group clearly applies a deterministic utility model and preserve the all-or-nothing property assuming that the facilities are either identical or different in terms of their attributes. The papers employing the probabilistic customer allocation employ random utility models, where each customer splits his/her demand into multiple facilities. All papers with probabilistic allocation utilize the gravity-based model with the following exceptions. The paper by Drezner et al. [20] is classified as "Proportional" since in that work customers may be attracted to multiple facilities if they remain within the region of influence of a facility and their purchasing power is distributed equally among all capturing facilities. Furthermore, the market share captured by a firm from a customer is proportional to the number of its facilities attracting that customer and inversely proportional to the total number of attracting facilities belonging to both the firm and its competitors. Besides [20], Biesinger et al. [12] also employ a proportional allocation rule in addition to deterministic as well as gravity-based rules. In this work, each customer is attracted to the nearest facility of the leader and follower at the same time and his/her demand is proportionally split between the two facilities according to their attractiveness levels, which only depend on the distance from the customer.

The seventh and eighth columns of Table 1 give information on the number of facilities opened by the leader and the follower: they either open a single facility ("S"), or multiple facilities ("M"), or do not locate any new facility ("N") because their decisions are only related to the redesign of existing facilities. The last column labels the reviewed papers with respect to the solution approach adopted. Two classes of studies are identified: those that are solved by a heuristic method ("H") to find a good feasible solution for the leader, and those that are solved by exact methods ("E") to determine an optimal solution of the leader.

Most of the survey papers on CFL problem group the reviewed works according to various components such as candidate locations, the number of facilities to be opened, the existence of reaction, and the objective function. In this chapter, we select two major ingredients of the CFL problems that include a BP model: leader's decision(s) and the allocation of customers to the facilities. In our opinion, these two issues are important because of two reasons. First, they have a hinge on how realistic the models are. Second, they specify the difficulty level of the developed BP model. Therefore, we partition the papers given in Table 1 into four sections. Sect. 2 and Sect. 3 include the papers where the leader decides only on the location of new facilities. The difference among the papers in these two sections is that Sect. 2

Table 1 Characteristics of the reviewed papers

	Leader's	Follower's	Location	Allocation		No. of Facilities		Solution
	Action	Reaction	Type	Deterministic	Probabilistic	Leader	Follower	Method
Alekseeva et al. (2015)	L	L	Disc	Dist		M	M	E
Arrondo et al. (2014)	L+D	L+D	Cont		Gravity	S	S	H
Ashtiani et al. (2013)	L	L	Disc		Gravity	M	M	–
Beresnev (2009)	L	L	Disc	Pref		M	M	H
Beresnev (2012)	L	L	Disc	Pref		M	M	H
Beresnev (2013)	L	L	Disc	Pref		M	M	E
Beresnev & Mel'nikov (2011)	L	L	Disc	Pref		M	M	H
Bhadury et al. (2002)	L	L	Cont	Dist		M	M	H
Biesinger et al. (2014)	L	L	Disc		Gravity	M	M	H
Biesinger et al. (2015)	L	L	Disc	Dist		M	M	H
Biesinger et al. (2016)	L	L	Disc	Dist	Gravity/Prop	M	M	H
Davydov et al. (2014a)	L	L	Cont	Dist		M	M	H
Davydov et al. (2014b)	L	L	Disc	Dist		M	M	H
Drezner and Drezner (2002)	L	L	Cont		Gravity	S	S	H
Drezner et al. (2015)	L+D	L+D	Disc		Prop	N/M	N/M	H
Fischer (2002)	L+D	L+D	Disc	Price		M	M	H
Hendrix (2015)	L+D	L+D	Cont		Gravity	S	S	E
Kochetov et al. (2013)	L+D	L+D	Disc		Gravity	M	M	H
Kononov et al. (2009)	L	L	Disc	Pref		M	M	E
Küçükaydın et al. (2011)	L+D	D	Disc		Gravity	M	N	E
Küçükaydın et al. (2012)	L+D	L+D	Disc		Gravity	M	M	E/H
Mel'nikov (2014)	L	L	Disc	Pref		M	M	H
MirHassani et al. (2015)	L	L	Disc	Dist		M	M	H
Panin et al. (2014)	L	L	Disc	Price		M	M	H
Plastria and Vanhaverbeke (2008)	L	L	Disc	Dist		M	S	E
Rahmani and Yousefikhoshbakht (2012)	L	L	Disc	Pref		M	M	–
Ramezanian and Ashtiani (2011)	L	L	Disc		Gravity	M	M	E
Redondo et al. (2010)	L+D	L+D	Cont		Gravity	S	S	H
Redondo et al. (2013)	L+D	L+D	Cont		Gravity	S	S	H
Sáiz et al. (2009)	L	L	Disc		Gravity	S	S	E
Shiode et al. (2009)	L	L	Cont	Dist		S	S	E
Shiode et al. (2012)	L	L	Cont	Dist		S	S	E
Tóth and Kovács (2016)	L	L	Netw		Gravity	S	S	E

contains papers with deterministic customer allocation, whereas Sect. 3 consists of the studies with probabilistic allocation. The papers with location and design decisions of the leader are discussed in detail in Sect. 4 and Sect. 5. These two sections are differentiated from each other again by the type of the allocation of customers to facilities.

2 Leader's Decision: Location, Allocation: Deterministic

Alekseeva et al. [1] consider an $(r|p)$-centroid problem in the discrete space. Since the facilities in the market are assumed to be identical, the customers visit only the closest open facility, where ties are broken in favor of the leader. The authors propose an iterative exact method which makes use of a single-level reformulation including polynomial number of variables and exponentially many constraints. At each iteration of the method, a family of the constraints representing the follower strategies are extracted and the best strategy of the leader against this family is found by a local search heuristic, which provides an upper bound on the objective function value of the leader. The algorithm stops when the upper bound is equal to a lower bound. To this end, the family of constraints is enlarged at each iteration.

Two different settings are developed in Beresnev et al. [5] for a CFL problem in discrete space with a deterministic customer choice rule. The two settings differ from each other in terms of the LLP. In the first setting, both players try to maximize their profit gained from customers, whereas in the second setting fixed cost of opening facilities is subtracted from the income captured by follower's facilities which represents the number of customers. Moreover, the facilities opened by the follower are not allowed to be detrimental, i.e., the obtained income should exceed the sum of the fixed costs. The leader and the follower share the same set of candidate facility sites, but the follower cannot open its new facility at a site occupied by the leader. Each customer shows his/her preferences towards the new facilities by considering a linear order. Then they select the first open facility according to this order. The author proposes an algorithm which constructs an auxiliary pseudo-Boolean function, called estimation function, whose minimum value is sought to obtain an upper bound on the objective function value of the bilevel models.

Beresnev [6] considers a sequential game between two competing firms that share the same set of candidate facility sites. Each customer is served by a single open facility and is captured by a leader's facility if it is the most desirable one among all open facilities of the leader. However, it is assumed that the customers who do not patronize any of the leader's facilities can be captured by any of the follower's facilities as long as it is more desirable than all of the leader's facilities. A bilevel integer linear programming model is introduced to represent the problem. Since the presence of multiple optima in the LLP poses uncertainty, the author introduces two rules, namely *the rule of cooperative behavior* and *the rule of non-cooperative behavior*. When the rule of cooperative behavior is imposed, the follower chooses the optimal solution to its problem which provides the best outcome for the leader. On the contrary, when the rule of non-cooperative behavior is assumed, then the follower selects the optimal solution which decreases the objective function value of the leader the most. In order to obtain approximate cooperative and non-cooperative solutions, an algorithm that consists of two stages is proposed. At the first stage, a solution of the leader is fixed and the optimal objective function value of the follower is estimated. Then at the second stage, an auxiliary problem is solved, which provides the desired feasible solution of the bilevel problem. In order to obtain these feasible solutions, an algorithm for estimating an upper bound on the objective function value of the bilevel model at any feasible solution is developed. The author represents the problem to find the optimal cooperative and non-cooperative solutions as the maximization of pseudo-Boolean function which is solved by a local search algorithm.

Beresnev [7] is concerned with the same bilevel integer linear programming problem given by Beresnev [6]. In contrast to the study of Beresnev [6], the author suggests a branch-and-bound (B&B) method to find the optimal non-cooperative solution. The problem at hand is first converted into a problem of maximizing a pseudo-Boolean function, where the number of variables coincides with the number of candidate facility sites of the leader. Since this function is implicitly defined, first the LLP and then the auxiliary problem should be solved to optimality. The author adopts the same method used in [6] for estimating an upper bound. In [6], the upper bound is generated for the case where the profits obtained from customers are the same for all facilities. However, in this study the algorithm produces an upper bound in the general form and provides at the same time a feasible solution which yields a lower bound. In order to obtain a good incumbent solution at the root node of the B&B tree, a standard local search algorithm is used which is devised in [8] dealing with a similar problem in [5] and [6].

A centroid problem in the plane is dealt with in Bhadury et al. [9], where the follower locates additional facilities as a reaction. Both players wish to capture most of the demand that is represented by varying the weights at discrete points. The customers choose the closest facility and in case of a tie, leader's facility is preferred. The authors present two heuristic methods to solve the medianoid problem, namely a greedy heuristic and a minimum differentiation heuristic which turns out to be more robust. The latter algorithm is based on an observation of Hotelling: when the prices are fixed and equal, the facilities tend to be located at a central point in the market under duopoly. The idea behind this is basically to locate a new facility at an arbitrarily small distance away from an existing facility. After the medianoid problem is solved with these two heuristics, they make use of them again to solve the ULP. When a solution of the leader is fixed, the follower's problem is solved. Given this solution of the follower, the leader acts like the follower and its problem is solved using the same two heuristic methods. This alternating procedure is repeated until a stopping criterion is met.

Biesinger et al. [11] formulate a bilevel mixed-integer linear model for a discrete $(r|p)$-centroid problem on a weighted complete bipartite graph. Only one facility can be opened at a predetermined candidate site, where each customer is assigned to the closest facility. If there are two closest facilities to a customer, he/she chooses the one which belongs to the leader. A hybrid genetic algorithm is proposed to solve the problem. In order to evaluate a solution of the leader, three methods are considered, which leads to a multi-level evaluation scheme. The first one includes an exact evaluation by solving the follower's problem by means of a commercial solver. The second method solves the LP relaxation of the follower's problem which provides a lower bound on the objective function value of the leader. Finally, the last method employs a greedy algorithm on the follower's problem to get an upper bound on the objective function value of the leader. The best result is obtained by the second method. The proposed genetic algorithm builds up a solution archive for identifying solutions that have been already generated. The generated solutions are stored in a special data structure and the duplicates are converted into new solutions. The algorithm is then locally improved using a tabu search procedure. The best computational results are obtained by the algorithm using the tabu search with a reduced neighborhood. Davydov et al. [16] are concerned with the same discrete $(r|p)$-centroid problem considered in [11]. This time the problem is solved by two heuristic methods, namely a local search with variable neighborhood search and a stochastic tabu search method. Both procedures employ a neighborhood swap over leader's variables. To accelerate the local search, the neighborhood is partitioned into three parts and the two most promising parts are searched thoroughly by finding the ascent direction quickly.

An $(r|p)$-centroid problem in the continuous plane is addressed in Davydov et al. [15] for identical facilities, where the customers select the closest facility. The follower's problem is reformulated as an integer linear programming problem that describes a maximal covering problem introduced by Church and ReVelle [13] and solved exactly by a B&B method. In order to solve the centroid problem, the authors develop a local search heuristic based on variable neighborhood search. In order to find the best neighboring solution according to the swap neighborhood, the $(r|X_{p-1}+1)$-centroid problem is considered as a subproblem in which it is assumed that the leader has already opened $p-1$ facilities and wishes to locate one more additional facility. Moreover, it is shown that the $(r|X_{p-1}+1)$-centroid problem is polynomially solvable for fixed r.

In the CFL problem analyzed in Kononov et al. [31] customers choose a facility to visit on the basis of their preference over a facility. By assuming that the preference h_{ij} of customer j is different for each facility site i, the authors avoid ties between two or more facilities, which eliminates the need to consider optimistic and pessimistic strategies. As a matter of fact, it is not the absolute preference values but their relative values or ranking, which determines the assignment of each customer to a facility. Hence, a deterministic allocation model is developed. Both the leader and the follower open multiple facilities among a set of candidate sites to maximize the profit giving rise to a discrete CFL model. The problem is solved approximately by successively generating upper bounds on the optimal value of the leaders objective function and lower bounds on the optimal value of the followers objective function. Mel'nikov [34] tackles essentially the same problem considered in [31] except a customer may have the same preference for two different facility sites. A randomized local search algorithm is proposed in the paper for the solution of the problem. The same problem is also studied in MirHassani et al. [35] where customers patronize the facility closest to them. In other words, instead of the preference matrix h_{ij}, distance matrix d_{ij} is prepared among the customer locations and candidate facility sites. As the solution approach, the authors utilize a modified quantum binary particle swarm optimization (QBPSO) method that uses an improvement procedure within the QBPSO. The benefit of the improvement is increasing the speed of convergence and preventing the algorithm from being trapped in local optimal solutions. The CFL problem analyzed in Panin et al. [37] differs from the others in this group in one aspect: the allocation of customers to facilities. Rather than taking into account the distance between the customer location and facility site or a predetermined preference parameter as is the case in [31, 34, 35], customers choose the facility to be visited on the basis of the selling price of the product set by the players differently for each customer. Associated with customer j, there exists a maximal price w_j that this customer is willing to pay for the product. Hence, the firms cannot set a price larger than w_j. Furthermore, there is a cost c_{ij} which represents the cost of serving customer j from an open facility at site i, which constitutes a lower bound on the price. Finally, the number of facilities to be opened by the leader and follower firms is fixed as in the well-known p-median problem as opposed to the other aforementioned papers which treat the number of facilities opened an endogenous variable, i.e., determined by the solution of the model.

The CFL problem analyzed in Plastria and Vanhaverbeke [39] is based on the maximal covering model. The aim of the leader is to maximize the demand covered by its newly opened facilities by taking into account the market entry of a competitor with a single facility. The allocation of customers is modeled using the so-called patronizing sets associated with customer j, namely customer at location j can be served by a facility that is opened within a certain distance away from the customer. Candidate facility sites within this region belong to the patronizing set S_j of customer j. An important assumption regarding the allocation is that if both the leader and follower open a facility in the patronizing set of a customer, then this customer is served from a leader's facility. Although this is a Stackelberg game that can be formulated as a bilevel model, the authors propose a single-level mixed-integer programming model, which is solved using ILOG CPLEX 9.0.

In Rahmani and Yousefikhoshbakht [40] the authors consider a closed-loop supply chain network where customers not only have demand for new products, they also want to return used products. Both players can open one of three kinds of facility at a candidate site: forward, backward, and hybrid processing facility. Moreover, multiple facilities of a given type can be established by both parties. A bilevel mixed-integer nonlinear programming model is developed, which is then transformed into bilevel mixed-integer linear model. No attempt has been made to solve the resulting model.

Shiode et al. [46] deal with the following variant of the CFL problem. Each firm opens a single facility in the continuous plane. Customers choose the nearest facility to them. The feature differentiating the problem from those studied in other papers is that customer demand is dependent on whether the facility belongs to the leader or follower firm. Furthermore, the distance between customers and facilities is measured by the rectilinear or rectangular distance. It is shown that in the case of linear market, where all customers are located on a line, the optimal location for leader's facility coincides with a demand point. In the case of planar market, where customer locations are scattered in the plane, leader's facility is optimally located on one of the grid points that are obtained by drawing horizontal and vertical lines through the customer locations.

Shiode et al. [47] develop a trilevel model rather than a bilevel one because there are three competitors which try to find the best locations for their single facilities. The demand of customers are assumed to be continuously distributed along the line segment and each customer visits the nearest facility. In fact, it turns out that there are infinitely many customers existing along the line segment, but the continuously distributed demand in an interval has a finite value.

3 Leader's Decision: Location, Allocation: Probabilistic

Ashtiani et al. [3] suggest a discrete CFL problem, where the leader wishes to determine the optimal location for p new facilities to maximize its market share. The follower, on the other hand, wants to maximize its market share by locating r new facilities after leader's action, but r is uncertain to the leader. It is assumed that the follower can locate either $1,2,\ldots,r$ new facilities at candidate sites where each number corresponds to a different scenario. The only information that the leader obtains is the probability of occurrence for each scenario. Since Huff's gravity-based rule is employed, the existing and new facilities have various attractiveness levels. However, the attractiveness level of a new facility is not considered as a decision variable in the model; they are rather predetermined for each candidate facility site. Since the exact number of new facilities of the follower is not known to the leader, robust optimization is employed for the solution of the problem, where the objective function of the leader maximizes the expected value of the market share under various scenarios and minimizes the difference between the optimal solution of a scenario and the expected value. Although the authors claim that the leader's problem is solved to optimality, no solution methodology is given.

Six different bilevel models in discrete space including only location decisions are considered by Biesinger [12]. The various models are based on three customer choice rules: binary, proportional, and partially binary. In a binary model, customers visit only the closest facility, whereas the proportional model considers Huff's gravity-based rule. In partially binary case, the demand is distributed among the closest facilities of the leader and the follower, where the attractiveness depends on the distance between the customer and the facility. Although the attractiveness levels of the facilities are taken into account for the proportional and partially binary cases, they are set equal to one so that the utility of a facility for a customer relies solely on the distance. Each of these customer choice rules is combined with both essential and unessential demand which gives rise to six different bilevel models. In case of essential demand, the entire demand of customers is satisfied, whereas a certain proportion of customers' demand is fulfilled based on the distance to the serving facility for the unessential demand case, i.e., the demand decreases with the increased distance. The bilevel model applying the binary customer choice rule with essential demand coincides with the model by Biesinger [11] given in Sect. 2. The LLP is formulated as linear mixed-integer programming problem in all scenarios which can be solved by the same three methods suggested by [11]. Again an evolutionary algorithm is further improved by a complete solution archive and is turned into a hybrid procedure by employing a tabu search variant which uses the solution archive as the tabu list.

Biesinger et al. [10] suggest the same bilevel model using the proportional rule with essential demand considered in Biesinger [12], where the attractiveness levels of the facilities are equal to one. The follower's problem is first converted into a mixed-integer problem following a linear transformation. An evolutionary algorithm with a complete solution archive and an embedded tabu search method to optimize leader's locations is employed.

Drezner and Drezner [19] propose three models which take changing market conditions over a planning horizon into consideration. Only the second proposed model, namely the Stackelberg equilibrium model, is discussed here. Both players want to maximize the market share by locating a single facility in the continuous plane. Huff's gravity-based rule is employed to estimate the market share. However, only the location of the facilities is sought. It is assumed that the follower opens its single facility at some time point t_1 over the planning horizon t. It is further assumed that the buying power of customers can change in time, but the facility attractiveness not. In order to solve the problem, the authors suggest three heuristic procedures, namely a brute force approach, a pseudo-mathematical programming approach, and a gradient search approach, but no computational results are reported.

Ramezanian and Ashtiani [41] deal with the version of the CFL problem in which the leader wants to open p new facilities among predefined candidate locations by anticipating that the follower will react by installing r new facilities. Both players are assumed to have already existing facilities. By making use of the Huff's gravity-based model, the authors try to find the optimal facility locations of both parties by an exhaustive enumeration method, where the objective function of the leader and follower is to maximize their own market share. As a consequence, only a small problem instance is solved consisting of 16 demand points and 5 existing facilities three of which belong to the leader while the remaining two are owned by the follower. Parameters $p = r = 2$ implying that two new facilities are opened by the competitors.

Virtually the same problem described above is investigated in Sáiz et al. [45] with $p = r = 1$. The distinctive feature of this paper is the solution approach, which is a branch-and-bound algorithm. Two B&B algorithms are developed: one for the follower's lower-level problem and one for the leader's upper-level problem. The main idea in this algorithm is recursive partitioning of the original problem into smaller disjoint subproblems until the solution is found. It is guaranteed that a global optimum is found by successively generating new lower and upper bounds that ultimately lie within a given interval.

Tóth and Kovács [49] examine also a similar CFL problem with two differences. First, the problem is formulated on a network where customers represent nodes and facilities can be located along the edges of the network in contrast to the two papers cited above that allow facilities to be opened at candidate sites. Second, operational costs are taken into account for the single facility opened by the leader and the follower. Due to this cost component, the zero-sum property in the objective function, which is used in the B&B algorithm in [45], is violated and hence a more difficult problem is obtained. The authors devise a B&B method to solve the leader's problem that includes another B&B embedded within the former. The calculation of lower and upper bounds involve interval arithmetic and DC (Difference of Convex functions) decomposition.

4 Leader's Decision: Location and Design, Allocation: Deterministic

The only study that belongs to this category is due to Fischer [24]. This study focuses on a discrete Stackelberg-type CFL problem with two competitors. The customers are aggregated at discrete points which make up the markets and each competitor supplies the same product to these markets. Both the leader and the follower want to decide on the locations of a fixed number of new facilities and the price of the product at each market. Since the price at a market is defined by the distance between the facility and the market, the price of the product can differ from market to market (i.e., discriminatory pricing). It is assumed that customers buy the product from the competitor giving the lowest price. Two bilevel models are formulated: a mixed-integer nonlinear bilevel model in which both players fix their locations and prices in the end, and a linear bilevel model with binary variables where price adjustment is possible. A heuristic solution procedure is developed to solve the linear bilevel model, but no computational result is given.

5 Leader's Decision: Location and Design, Allocation: Probabilistic

Hendrix [26] considers a problem where both the leader firm and the follower firm want to open a single facility in the continuous plane. In addition to the location of their facilities, the players also aim to determine the facilities' quality level. The objective functions are represented by the respective profits of the firms, where opening a facility with quality x incurs a linear cost cx. The choice of each customer is modeled using Huff's gravity-based rule based on the ratio of the quality to the Euclidean distance between the location of the customer and facility site. One of the important results is that co-location of the leader's and follower's facilities is not possible. The authors also give conditions under which one of the firms can force the other not to enter the market due to the negative profit.

In Küçükaydın et al. [32], the authors deal with a variant of the CFL problem where multiple facilities are opened by the leader firm among a predetermined set of candidate sites to maximize its profit. The quality levels of these facilities are also decision variables. The reaction of the competitor firm, which is the follower, consists of adjusting the quality of its existing facilities. Employing Huff's gravity-based rule, the authors develop a bilevel mixed-integer nonlinear programming (MINLP) model and solve it using a global optimization method called GMIN-αBB after converting the bilevel model into an equivalent one-level MINLP model. This conversion is possible thanks to the concavity of the objective function of the follower's lower level problem in terms of the quality variables when the leader's decision variables are fixed.

Küçükaydın et al. [33] extend the work in [32] by adding the capability of opening new facilities and/or closing existing one to the competitor's reaction set. Clearly, the new problem is a more challenging one. First, an exact solution procedure is developed that integrates complete enumeration in terms of competitors location variables for opening new facilities and keeping/closing existing facilities with GMIN-αBB. Since the exact method can only provide solutions for small problem instances in a reasonable CPU time, three heuristics based on tabu search are proposed. These heuristics perform a search over the location variables of the leader in the upper level problem. Upon fixing these variables, the follower's lower level problem is solved to optimality using a branch-and-bound algorithm with NLP relaxation because it is shown to be concave optimization problem.

In the CFL problem examined in Redondo et al. [42] the leader firm wants to open a single facility in the continuous plane in addition to its existing facilities in the market. The quality of the new facility is also a decision variable to be set by the firm. The reaction of the follower firm consists of exactly the same decisions as the leader in the sense that the location of a new facility and its quality level has to be determined. This problem is also referred to in the literature as the $(1|1)$-centroid problem. Four heuristics are developed for the solution of the problem. These are a grid search procedure, an alternating procedure and two evolutionary algorithms. The only difference between the problem investigated in Redondo et al. [43] from the one in [42] is that the demand of customer j is not fixed, but varies according to the utility the facility provides to the customer, which is computed according to Huff's gravity-based model. The variable demand also referred to as elastic demand of a customer is assumed to vary between the minimum possible demand and maximum possible demand. Note that most of the literature on CFL uses the assumption of inelastic or fixed demand. In order to solve the medianoid problem, the authors apply an exact interval branch-and-bound method and an evolutionary algorithm called UEGO that uses a Weiszfeld-like procedure and can find the global optimum with a certain reliability. Then, they develop three heuristic methods to solve the centroid problem, i.e. leader's problem: a grid search procedure, a multi-start algorithm, and a subpopulation-based evolutionary algorithm, called TLUEGO. The same problem is then considered by Arrondo et al. [2] to improve the computational performance of TLUEGO. To this end, they suggest three parallelizations of the algorithm, namely a distributed memory programming algorithm, a shared memory algorithm, and a hybrid of these two algorithms.

Drezner et al. [20] introduce a BP model for a discrete CFL problem taking into account the concept of cover. Each existing and new facility possesses a *radius of influence* which defines a threshold distance to the customers, i.e. a customer visits a facility if and only if his/her distance to that facility is less than or equal to the radius of facility. Hence, a facility can capture only the customers who are within its *sphere of influence*. Thus, the radius of each facility determines its attractiveness level. The demand of a customer who is not covered by the sphere of influence of any facility is considered lost. The leader and follower aim to maximize the market share under a certain budget by expanding their own chains conceiving three strategies. The first strategy takes into account only enlarging the sphere of influence of the existing facilities, whereas the second one solely opens new facilities whose location and radii need to be determined. The final strategy combines the first two strategies. The authors define the radius of the facilities as a continuous decision variable. However, they employ discrete design (radius) scenarios in the solution of the model and one of a finite number of available radii is determined for each open facility. First, the follower's problem is solved to optimality using a B&B method. Then a tabu search algorithm is implemented for the solution of leader's problem which uses a greedy-type heuristic to find a starting solution. The computational results indicate that both the leader and follower can extend their market share by acquiring the lost demand.

Kochetov et al. [30] propose a discrete CFL bilevel problem making use of the Huff's gravity-based rule. Both the leader and follower want to maximize their market share by deciding on the location and attractiveness for their new facilities under a budget limitation. As in the study of Drezner et al. [20], a discrete design scenario set is adopted for the attractiveness levels of the facilities. They apply a linear transformation on the follower's problem to turn it into a linear mixed-integer programming problem. Then an alternating matheuristic, derived from Bhadury et al. [9], is employed for the solution of the bilevel model which terminates when either a Nash equilibrium is reached or an already visited solution is obtained.

6 Concluding Remarks

In this chapter, we review and categorize studies on CFL problems formulated as BP models which are published between 2002 and 2016. We first categorize each study according to various features of the CFL problem investigated. After collecting each work in one of the four groups depending on two distinguishing factors of CFL problems, namely the leader's decision(s) and the allocation of customers to facilities, we give detailed information on each work. It must be noticed that the studies in which decisions are made not only about the locations of the facilities but also about their design is outnumbered by the papers containing only location decisions. Thus, future studies can give more emphasis to this issue. Another observation is related to the fact that relocation, redesign, and/or closing of existing facilities concurrently with opening new facilities can be studied more thoroughly with the inclusion of additional features incorporated into the CFL problem. Another fruitful research direction can be the extension of bilevel models in a setting where multiple firms acting simultaneously among themselves react to the leader. Such a setting introduces the requirement of considering Nash games in the lower-level problem of a bilevel programming model.

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