Characterizing a highly-accomplished teacher's instructional actions in response to students' mathematical thinking

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This paper is part of a larger study which investigates how a highly-accomplished teacher and two beginning teachers notice student thinking and respond to students' mathematical thinking as they teach concepts of multiplication and division in a third-grade classroom. The focus of this paper is on describing highly-accomplished teacher's instructional actions in response to student thinking which are different than that of the beginning teachers. The participant teachers' instructional actions were analysed utilizing a framework developed by Cengiz, Kline and Grant (2011). The results revealed that the highly-accomplished teacher challenged student thinking with counter arguments and introduced alternative representations more frequently, but complimented students less frequently than the beginning teachers.

Keywords: Instructional actions, responding to students' mathematical thinking.

INTRODUCTION

Creating instruction based on student understanding and thinking of mathematics is one of the essential practices underlying teaching for understanding (Ball, Lubienski, & Mewborn, 2001; Fennema, Franke, Carpenter, & Carey, 1993). Research shows that attending to mathematical thinking of students in professional development programs can help improve both teaching quality and student achievement (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Kazemi & Franke, 2004). Although these findings establish the importance of mathematics teachers' understanding of student thinking and their pedagogical decisions based on this understanding, responding to student thinking in appropriate ways is a complex skill, which requires hearing and interpreting student thinking (Wallach & Even, 2005). Therefore, there is a need to understand this core practice of teaching and help teachers improve in their practices.

Due to the interactive and clinical nature of teaching, Grossman and McDonalds (2008) call for studies that investigate details of teachers' practices instead of their knowledge or beliefs. Grossman and McDonalds (2008) argue that research in teaching lacks "common pedagogies for helping novices learn to respond to student thinking in the moment," even though teachers' responses to students during interactive teaching is one of the major components of the teaching practice.

Berliner (2001) identified flexibility and adapting the lesson according to students' responses as one of the major skills that differentiate experts from novices. Among other studies, Borko and Livingston (1989) provided further evidence for this argument by observing and interviewing student teachers and their cooperating teachers before and after they taught lessons for a week. Researchers found that the cooperating teachers were much better than their student teachers at improvising the lesson according to unexpected student questions or comments.

There are relatively few studies that specifically focus on how teachers respond to student thinking during mathematics instruction (Even & Gottlib, 2011; Pierson, 2008). While many studies document different ways the way teachers respond to student thinking, these studies are either focused on classroom discourse (Even & Schwarz, 2003), teacher practices as they implement a specific curriculum (Fraivillig, Murphy, & Fuson, 1999), as part of an intervention program (Doerr, 2006), or in the context of a professional development program outside classrooms (Jacobs, Lamb, Philipp, & Schapelle, 2011).

Cengiz, Kline and Grant (2011) investigated how six elementary school teachers elicited, supported and extended students' mathematical thinking through classroom observations and interviews. The authors developed a framework building on the work by Fraivillig and colleagues (1999) that focused on instructional actions related to extending student thinking during whole class discussions. The authors conceptualized extending student thinking as "helping students move beyond their initial mathematical observations and further develop an understanding of a mathematical phenomenon" (p. 356). The most common instructional actions of extending student thinking were grouped under categories of encouraging mathematical reflection, going beyond initial solution methods, and encouraging mathematical reasoning. Using counter-speculation and introducing representations and contexts that are familiar to students were the least frequently observed instructional actions that supported or extended student thinking.

The purpose of this study is to describe characteristics of a highly-accomplished teacher's instructional actions, specifically in response to students' mathematical thinking. This kind of research may offer insights in teachers' learning as well as helpful learning experiences for novice teachers in improving their practices of building instruction on students' mathematical thinking. In this study, I prefer to use the term *highly-accomplished teacher* rather than expert teacher because the latter is not clearly defined and has many different connotations (Li & Kaiser, 2011). I draw upon Schoenfeld's (2011) definition of a highly-accomplished teacher as one who spends minimal time on classroom management issues and engages in diagnostic or responsive teaching most of the time.

METHOD

The case study methodology was used in the current study. Using this methodology allows the researcher to answer questions such as "how?" and "why?" while considering the influence of context on a phenomenon within which it is situated (Baxter & Jack, 2008). The purpose of the larger study (Taylan, 2013) is to characterize third-grade teachers' practices of noticing of student thinking and instructional actions in response to student thinking during teaching. Because the larger study explores noticing of children's mathematical thinking and teaching practices among three different third-grade teachers, one experienced and two in their first-year, it represents a multiple-case study which allows the researcher to explore differences and similarities within and among cases (Yin, 2003). The findings in this paper focus on the case of the highly-accomplished teacher.

The researcher observed and videotaped each teacher's mathematics classes for a week. Each third-grade teacher worked on topics of multiplication and division in the same school district during the time of data collection. The teachers wore portable cameras and selected moments of their instruction as they taught and reflected on the selected video clips during the interviews that followed each class. The interviews and teacher-selected video clips provided additional insights on the teachers' instructional decision making processes. The researcher's observation notes and transcribed video observations of each class together with teacher interviews and lesson plans allowed the researcher to have a robust understanding of teachers' instructional actions as they responded to student thinking in the context of each classroom.

Selection of case and background

Brooke was nominated for participation as a highly-accomplished teacher by the district mathematics coordinator and school principal, both of whom have observed her teaching before. Brooke has been teaching third grade for 6 years and she has 3 years experience of attending a contextualized and an intensive professional development program where she worked one-on-one with a prominent teacher educator and educational researcher.

Brooke taught at a school where many students came from low socio economical backgrounds. With regards to mathematics instruction in her classroom, Brooke aimed for her students to comfortably share their mathematical thinking and "think for themselves" (Brooke, the first interview) instead of seeking her approval. Brooke believed her students needed multiple types of opportunities in order to understand the content she planned to introduce.

The research project took place when Brooke's class was making a transition from learning multiplication to division. The activities that Brooke created and planned together with her colleagues involved different models of multiplication and division: students jumping on the number line taped on the floor to represent skip-counting situations, and cutting ribbons of equal length for wrapping gifts. Brooke described her teaching goals in the following way:

> One of our district objectives right now is having a variety of models for division and multiplication and make connections between multiplication and division and so we have done like discrete models and then we are supposed to introduce number line models. For some of my kids that have difficulties with number sense number lines are helpful, which we did in multiplication. Some of the things they used for multiplication might help them solve division problems more efficiently rather than having to repeatedly subtract and be inaccurate in their computation (Brooke, the first interview).

Brooke aimed to create meaningful experiences for her students so that they could make connections between the new topic of learning division and what they already knew in multiplication.

Data Analysis

Instructional actions examined in this study consisted of responses to student thinking (any spoken or written mathematics related ideas, justifications or generalizations) exhibited by teachers during instruction (Fogarty, Wang, & Creek, 1983). The nature of instructional actions as teachers responded to students' mathematical thinking was investigated by analyzing whole-class videos of classroom observations and field notes. Classroom talk pertaining to each lesson was transcribed, and instructional actions were analyzed only when they occurred as a response to student's mathematics related question, answer, comment or claim.

Guided by the frameworks of Fraivillig and colleagues (1999), Pierson (2008) in general and, Cengiz and colleagues (2011) in particular, teachers' instructional actions were analyzed in two steps. First, chunks were identified that indicated existence of instructional actions responding to a student idea or question. Second, these chunks of teacher responses were subdivided into more detailed segments.

By using the conceptual framework shown in Figure 1, teachers' instructional actions were categorized as supporting instructional actions, extending instructional actions and others. To ensure reliability, classification of instructional actions within one lesson was also checked by another educational researcher until reaching an agreement about the coding scheme.

RESULTS

The results of this study emerged mostly through the use of the conceptual framework (Figure 1) and observing instructional actions across teachers for different instructional actions. Apart from the analysis of individual instructional actions, observation of a phenomenon that was emphasized in previous research, flexibility of an expert (or a highly-accom-

Supporting actions	**Other**
Repeating student idea, claim, question	*Complimenting or evaluating*
Suggesting an interpretation	*Clarifying questions*
Introducing different representations	*Requesting basic information*
Reminding of the goal	*Redirecting to a peer*
Recording student thinking	*Providing hints*
Acknowledging student thinking	
Extending actions	
Inviting students to evaluate a claim	
Inviting students to provide reasoning and probing	
Challenging/providing counter arguments to student claims	
Pushing for alternative ways	

Figure 1: Instructional actions framework (Adapted from Cengiz et al., 2011)

> Mrs. Paul has got this kid in her classroom named Sam. Here is the problem that she gave him: Mrs. Paul had 24 inches to wrap two presents. How long will each piece of ribbon be?
>
> I want you to look at Sam's work and then I am gonna ask you whether you agree or disagree with Sam and to tell me why. Alright so here is his work. Sam wrote 24 + 24 =48 and 24 x 2=48.
>
> If you agree with Sam's work you are going to go ahead and tell me why you agree. If you don't agree with Sam's work and his ideas, I want you to tell me why (Brooke, third lesson transcription).

Figure 2: A supplemental task after coming across a misconception

plished) teacher with regards to adapting instruction based on student thinking, deserved attention.

Although all three teachers provided evidence of noticing student thinking and employing instructional actions that supported student thinking, beginning teachers failed to introduce new tasks in modification to their lessons plans based on student strengths and weaknesses as they taught.

Brooke had the flexibility of changing the lesson plans by providing additional tasks she considered to be necessary. For instance, on the second day of observation some students used multiplication instead of division. On the third day, Brooke presented the following task given in Figure 2.

Most students who agreed with this misconception in the beginning of the lesson (more than half of the students in class) subsequently changed their thinking towards the end of the class as observed in student worksheets collected and teacher's assessment during interviews. Implementation of this task (Figure 2) could be considered as responding to student thinking based on the teacher's noticing.

In analysing classroom transcriptions of Brooke based on the analytical framework, several patterns emerged. First, Brooke exhibited instructional actions that had the potential to support and extend student thinking. In particular, *repeating a student idea*, *acknowledging student thinking* and *suggesting interpretation* were the most frequently used instructional actions that supported student thinking. With regards to instructional actions that had the potential of extending student thinking, the most frequently observed actions were *inviting students to provide reasoning and probing*, *inviting students to evaluate claims* and also *challenging/providing counter arguments*.

Brooke *introduced multiple representations of mathematical concepts* in her teaching and *challenged students* more frequently than the two beginning teachers did. On the other hand, Brooke did not *compliment student thinking* as frequently as the beginning teachers did in the study. Although most of the instructional actions were observed across both the highly-accomplished teacher's class and the beginning teachers' classes, some of the instructional actions almost solely occurred in Brooke's class. Therefore, it is important to provide details of how those instructional actions are enacted in order to understand the differences between the highly-accomplished and the two beginning teachers.

Challenging / providing counter arguments

Brooke challenged her students more frequently than did beginning teachers in this study, mostly by asking questions that helped students realize their own mistakes. Brooke specifically challenged her students in the first class, when she observed many misconceptions in her students' understanding of multiplication and division. Most of student misconceptions were revealed after Brooke introduced the task of finding the patterns between multiplication and division sentences. For example, the students worked on understanding the relationship between the two number sentences, such as 5 times 7 is 35, and 35 divided by 7 is 5. Several misconceptions came to surface during this discussion. For instance, some students thought that in division sentences, numbers are sequenced from the largest to the smallest number; such as in the example of 35, 7 and 5 (35 divided by 7 equals 5) while multiplication sentences go from the smallest to the largest numbers (such as 5 times 7 equals 35). Especially during the first lesson when Brooke observed that most students had this misconception, she challenged the students and provided counter explanations. The following excerpt from the first classroom observation transcription is a typical example of how Brooke challenged a student:

Student: I know that division starts with the big number and the first number in the multiplication sentence is the answer. And the multiplication goes from smaller to bigger.
Brooke: So is that pattern always true? What about this, what if I write 7 x 5 =35, is that number sentence true? Is it always gonna go from small to big? [*challenging, providing counter argument*].

As evident above, Brooke does not directly tell the student that his answer is problematic but instead she challenges his misconception by providing a counterexample in order for this student to arrive at this understanding himself. During the interviews Brooke noted that she avoided evaluative language such as "that is a wrong answer" because she wanted her students to think for themselves, independent of her approval or disapproval.

Introduction of multiple representations of mathematical concepts

Being aware of her students' weaknesses, Brooke used alternative representations to make the concepts more meaningful to them. Additionally, she believed that each student had a different way of learning and some models/representations made more sense than others to some students. Accordingly, she believed in using a variety of representations in her teaching, as evidenced in the following interview excerpt:

> The number line makes a lot of sense to some kids. It really helps them. For some kids it does not really make any sense. And some kids with the discrete model they are like "well, this is great." It makes complete sense, and for other kids it makes no sense. So sometimes that's why I like introducing a few ways to visualize it because I feel like different kids have different ways of thinking about it so it is nice to find something comfortable for them (Brooke, the first interview).

Further evidence of introducing different representations can be found in the following excerpt given below where Brooke helps two students by suggesting use of different representations when they have difficulties of cutting 45 inches of ribbon into 9-inch strips and finding the total number of strips.

Student: I know it is going to be 45 divided by 9 but I can't figure out the answer.
Brooke: Okay. What could you guys do, what tools could you use to figure that out? Could you use a meter stick or do you want to try number line that is erasable? [suggesting use of/introducing different representations] (Brooke, third lesson).

DISCUSSION

Analysis of the highly-accomplished teacher's instructional actions in responding to student thinking revealed distinct qualities. Although all teachers in the study worked towards building their instruction on student thinking via asking students to provide their reasoning, and restate and evaluate peer's thinking, the highly accomplished teacher's repertoire of instructional actions included challenging/providing counter arguments and introducing multiple representations, unlike novice teachers.

The highly-accomplished teacher's flexibility in building instruction based on her noticing of student thinking during teaching was a finding that was not surprising based on previous research findings (Berliner, 2001; Borko & Livingston, 1989). The task presented in Figure 2 was not included in Brook's weekly teaching plan. However, she developed and implemented this task based on what she believed students needed.

Some instructional actions were more common than others across all teachers. For instance, *inviting students to provide reasoning behind their answers and probing*, *repeating student answers*, and *inviting to restate peer's claims* were among the most common instructional actions that followed up student thinking. On the other hand, *challenging and providing counter arguments* to what students said was the least frequent instructional action for teachers. This result may not be surprising given the fact that it was observed rarely even among experienced teachers in Cengiz and colleagues' (2011) study. Lack of this particular instructional action, namely, *challenging and providing counter arguments* to what students said in beginning teachers' teaching was one of the most important distinctions compared to the highly-accomplished teacher's teaching. Challenging students' thinking with counter arguments is considered an important component of extending student thinking although it is not easy to come up with this type of arguments in the actual moment of teaching (Cengiz et al., 2011; Fraivillig et al., 1999). Being able to engage in this particular instructional action likely required Brooke to listen to students carefully and to be able to generate a counterexample in the moment that would confront their misconception by learning that it would not hold in every case. It may be the case that Brooke was able to challenge her students more frequently because her experience allowed her to develop certain schemata for providing responses when students had misconceptions (Borko & Livingston, 1989).

The instructional actions that require students to provide reasoning and evaluating peer's answers may not be enough to create discussions that would really benefit student learning if they do not take place during a well-planned math discussion. The selection and sequence of student ideas to be shared with the whole class are the key components to a quality instruction (Stein, Engle, Smith, & Hughes, 2008). The experienced teacher was confident in her choices as she selected particular students to share their thinking with the whole class and their sequence of appearance (Brooke, the third interview). This may be a difficult task for novice teachers.

Brooke *introduced different representations of the concepts* of multiplication and division frequently in her class. Brooke's experience in both teaching and professional development where she created curriculum materials and thought about appropriate models for student learning may have helped her to develop a larger repertoire of representations. According to Shulman (1986) being able to understand and present different representations of the same concept is an important component of teacher's pedagogical content knowledge.

CONCLUSION

Adapting instruction based on student thinking, challenging/providing counter arguments, and introducing multiple representations were characteristics of the highly-accomplished teacher's instructional actions in response to student thinking in this study. The findings have the potential to contribute to research in teachers' professional development, especially in creating schemata of instructional actions that novice teachers may learn from to become better teachers.

Providing illustrations of how experienced teachers employ a variety of instructional actions may prove valuable for novice teacher learning. Watching video cases of highly-accomplished teachers such as in this study, reading transcribed teaching or learning to provide hypothetical responses in the form of challenging students or providing multiple representations may be valuable tools for beginner teacher's learning of the profession of teaching.

There is a need for studies that investigate instructional actions of other experienced and highly-accomplished teachers as they teach different mathematical topics in different contexts. Considering future research, it may also be worthwhile to explore the relationship between specific instructional actions in response to student thinking and their impact on students' participation in mathematics discussions and achievement levels in mathematics.

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